
Inhomogeneities in the composition of planetary regoliths tend to be smoothed out by the effects of meteoroid impact. We discuss here a general approach for calculating the time evolution of vertical compositional profiles. Examples of such profiles might be silicate lag deposits on icy satellites [caused by sublimation (1) or sputtering (2)]; silicate-contaminated crater rays on a clean ice surface; or clean crater rays on a darker, contaminated surface; stable- and radio-nuclide distributions generated by cosmic rays (3); and implanted solar-wind ions on the lunar surface.

For many applications a parametrized relationship between crater depth $w$ and "turn-over" time $\tau$ may be sufficient. This is the approach used by Shoemaker et al. (4), Gault et al. (5), and others in discussing regolith mixing on the moon. We can anticipate, however, that occasionally it will be useful to ask the question, if a given depth profile has one shape at a certain time, what is its shape at a later time? Blake and Wasserburg (6) addressed this problem and derived an equation for the concentration of radio-nuclides as a function of time and depth. We discuss here a related approach, but one which is more general than theirs.

If the total amount of material of interest above a height $x$ at the point $(y,z)$ is $q(x,y,z)$, then we define an areal average $Q(x,t) = \int_{A} q(x,y,z)/A\,dz\,dy$, where $A$ is some reference area. Between times $t$ and $t + dt$, suppose that $n$ meteoroids strike $A$. The $i$th ($i = 1, 2, \ldots, n$) meteoroid produces a disturbed region $Q_i$ of area $a(w_i)$ and depth $\omega_i$. If for the $i$th impact, we define $q(x,y,z,t) = g(x_i,y_i,z_i,t)$, where $x_i,y_i,z_i$ is a local coordinate system, then

$$Q(x,t+dt) = [1 - \sum_{i=1}^{n} a(\omega_i)/A]q(x,y,z,t)_{\text{uncratered}} + \sum_{i=1}^{n} a(\omega_i)q(x_i,y_i,z_i,t)_{\text{cratered}}/A,$$

where $N_\omega$ is the gardening operator which produces the new concentration profile from the pre-existing one for an impact event of depth $\omega$. The preceding equation is clearly very complicated in general. It can be simplified by using the relation $q(x,y,z,t)_{\text{uncratered}} = Q(x,t)$, which is true for a large number $n$ of random impacts. Thus,

$$Q(x,t+dt) - Q(x,t) = (Zdt/n) \sum_{i=1}^{n} a(\omega_i)[q(x_i,y_i,z_i,t)_{\text{cratered}} - Q(x,t)],$$

where, since $n$ is large, we have used $n = ZA dt$. $Z$ is the total meteoroid flux.

To further simplify this equation, we make use of the result

$$\lim_{n \to \infty} \sum_{i=1}^{n} a(\omega_i)[q(x_i,y_i,z_i,t)_{\text{cratered}}]_{\sigma_i} / n = \lim_{n \to \infty} \sum_{i=1}^{n} a(\omega_i)[N_{\omega_i} q(x,t)]_{\sigma_i} / n.$$ 

This means that a suitably weighted sum of transformations of the actual grain profiles $g(x_i,y_i,z_i,t)$ is exactly equal (for large $n$) to the weighted sum of the same transformations of the average grain profile $Q(x,t)$. Thus, if we are interested in studying the evolution of the average quantity $Q(x,t)$, we can proceed with the calculation as if the regolith actually did have an areally uniform composition over the region $A$. Finally, defining a gardening operator $G$ which both transforms and locally averages the distribution $G_\omega[q(x,t)] = \int_{A(\omega)} dydz N_\omega[q(x,t)]/a(\omega)$,
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we have
\[ \partial Q(x,t)/\partial t = \int_{0}^{\infty} Z P(\omega) a(\omega) [G_{\omega}[Q(x,t)] - Q(x,t)] d\omega, \]

where \( P(\omega) \) is the probability density for cratering to a depth \( \omega \). This equation can also be written in a more compact form
\[ \partial Q(x,t)/\partial t = \int_{0}^{\infty} F(\omega) \partial / \partial \omega \ G_{\omega}[Q(x,t)] d\omega, \]

which is the fundamental gardening equation. The quantity \( F(\omega) \) has the dimensions \( l/\text{time} \), providing the basic time scale for turn-over to a depth \( \omega \), and is given by the expression
\[ F(x) = \int_{x}^{\infty} Z a(\omega) P(\omega) d\omega. \]

Caution is required in the interpretation of the solutions to the gardening equation. \( Q \) refers to profiles averaged over a reference area \( A \), which may be any terrain unit, such as a large crater ray system, etc. The data, which might be derived from Apollo core samples or Voyager television pixel elements, must be sufficient to generate an areally averaged composition distribution. There is no fundamental restriction on the size of the reference area. However, if \( A \) is too small, then the effects of large impact events (disturbed area \( \sim A \)) are not included in the above formulation. In this case, the gardening equation can still be useful, but \( t \) must be reset to zero, and a new initial distribution \( Q(x,0) \) established, after each such event.

In the uniform vertical mixing model of cratering, it is always possible to reduce the solution \( Q \) of the gardening equation to the evaluation of an integral. In this model the gardening operator is \( G_{\omega}[Q(x,t)] = x Q(\omega,t) \theta(\omega-x)/\omega + Q(x,t) \theta(\omega-x) \). For the artificial but illustrative initial condition \( Q(x,0) = f_0 x \theta(x-\omega_0) \), where \( f_0 \) and \( \omega_0 \) are constants, we find the explicit solution \( Q(x,t) = f_0 x [1 - \exp(-F(\omega_0) t)] \theta(\omega_0-x) + f_0 x \theta(x-\omega_0) \). The first term \( (x < \omega_0) \) of the solution could represent a (dark) contamination of an initially clean surficial ice layer. At small \( t \), this contamination is also small, and any sensing element of a high resolution probe would have a high probability of registering "white." This example illustrates the importance of sampling density for generation of a data base which can be compared with the calculated profiles.

Further development of the present approach to gardening can be anticipated in the following directions: (a) establishment of criteria for estimating fluctuations about the mean profile values discussed above; (b) inclusion of source and sink terms into the gardening equation (this seems to be straightforward); and (c) solution of the gardening equation with cratering models which go beyond the vertical mixing assumption.

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