Planetesimal formation by gravitational instability of a dust layer requires a non-turbulent solar nebula. Motions of the gas at more than a few cm/s would stir up the dust layer too much. Safronov (1) maintained that there was no energy source to drive turbulent motion. His conclusion has been challenged by others (2-4) who model the solar nebula as a turbulent, convective accretion disk. Self-consistent models of a convective disk depend on high opacity of the disk material, which must be provided by grains. Models using opacity coefficients that are functions of temperature only involve the implicit assumption that the dominant mass is in particles small compared to the relevant wavelengths of radiation. Coagulation of grains could render that assumption invalid. Grain coagulation is implied by the requirement of forming planetesimals, as the turbulent velocities (~1/3 sound speed) in the accretion disk would disrupt any dust layer. Planetesimal formation must commence during the turbulent stage; otherwise, by the time the disk became optically thin due to viscous spreading, it would not contain enough mass to form the planetary system. Two scenarios are possible: Collisional coagulation could form large planetesimals, simultaneously leaving a sufficient fraction of matter in small grains to maintain the nebula's opacity. Or, coagulation of grains into small (~cm) aggregates could lower the opacity enough for turbulence to decay. Larger bodies would then form by low-velocity collisions or gravitational instability in the quiescent disk.

I have calculated numerically the evolution of a population of grains in a turbulent solar nebula. The basic features of the model are described in (5). This is a 1-D calculation of coagulation and vertical transport. The disk is divided into discrete layers; in each, the grain size distribution is represented by the population in a series of discrete size bins. During each timestep, changes in the size distribution due to collisions are computed for each layer. Then particles in each size range are transported between layers by vertical mixing. In the nonturbulent case, the transport is downward only, due to settling toward the central plane. Turbulence allows the possibility of upward motion as well.

The vertical structure of the nebula is assumed adiabatic at r=30 AU, where the properties of the evolutionary model of (4) change relatively slowly with time. I assume central temperature 50 K, central density 4.8 x 10^10 g/cm^3, total surface density 470 g/cm^2. These parameters closely match those of the model (4). The opacity, \( \kappa_A \), is due primarily to ice grains. The expression of (4) is \( \kappa = 2 \times 10^{-17} T^2 \). I include grain size effects by assuming \( \kappa = 2 \times 10^{-17} (f/f_0) + (Q/4\pi) \Sigma d^n(d) \), where \( f_0 = 0.01 \) is the dust/gas mass ratio at time \( t=0 \), when all grains are assumed small. At any later time, \( f \) is the mass ratio considering only those grains smaller than \( \lambda = 0.2898/T \) cm, the wavelength of peak black-body flux at that temperature. The summation is taken over all size bins for which the particle diameter \( d > \lambda \); \( n(d) \) is the number density in each size bin. \( Q \) is an efficiency factor that depends on the IR absorptivity of the grains. In order for \( \kappa \) to be continuous at \( d = \lambda \), \( Q \) must vary with \( T \) (\( Q = 0.00387 T \) is merely an empirical fit over a limited range of \( T \)).

The turbulent velocity \( V_t \) of the gas is parameterized as a function of optical depth \( \tau \), \( V_t = (c/3)(\tau/(\tau + 1)) \), where \( c \) is the local sound speed. At each level, \( \tau \) is the optical depth of that level, plus that of all levels above it. The turbulence has a finite range of scales. The largest eddies are ~scale height \( H \) in size, with turnover time ~1/\( \Omega \). Their energy cascades...
through the spectrum of smaller eddies to a size where viscous dissipation is important. Dimensional arguments (6) give an energy dissipation rate $\epsilon \sim \frac{V_{t}^{3}}{H}$ erg/g/s. The smallest eddies have "inner scales": length $\lambda \sim (V_{t}^{3}/\epsilon)^{1/4}$, time $\tau \sim (V_{t}^{3}/\epsilon)^{1/2}$, velocity $u \sim (V_{t}^{3}/\epsilon)^{1/4}$. In the model nebula, there are -3.5 km, $10^{5}$ S, and 35 cm/s, respectively.

Turbulence gives the particles random motion with respect to the mean gas flow. Völkl et al. (7) express the rms velocity as a function of the ratio of the response time of the particle to gas drag, $t_{e}$, to the timescale of the largest eddies, $t_{k}^{-1/2}$. For $t_{e}/t_{k} << 1$, the particle velocity $\langle V_{d} \rangle$ governing turbulent diffusion is $-V_{t}$, while for $t_{e}/t_{k} >> 1$, $\langle V_{d} \rangle \sim V_{t}(t_{k}/t_{e})$. This result is valid for any $t_{e}$, if $t_{k}/t_{e} >> 1$. Völkl et al. also give an expression for the relative velocity between grains, which governs their collision rate: $\langle V_{r} \rangle \sim V_{t}(t_{e}/t_{k})^{1/2}$ for equal-sized grains. However, their derivation involves the assumption $t_{s} << t_{e}$, which is not correct for small grains. Grains with $t_{e}/t_{s}$ co-move with the gas in the smallest eddies, experiencing essentially a laminar shear flow. Laminar shear is much less effective than turbulence in promoting collisions; the effective relative velocity is $\langle V_{r} \rangle = \langle \text{particle diameter/shear rate} \rangle, -d/t_{s}$ (8). The expression for $\langle V_{r} \rangle$ greatly overestimates the collision rate for small grains; at $t_{e} = t_{s}(d \sim 100 \mu m)$, $\langle V_{r} \rangle \sim 10^{3}$. I use $\langle V_{r} \rangle$ for $t_{e} < 0.1 t_{s}$, $\langle V_{r} \rangle$ for $t_{e} > 100 t_{s}$, and their logarithmic average in the transition range.

Systematic motions become important for bodies large enough that $t_{e}/t_{k}$ is not negligible. Mass transport between levels is due to both systematic and random velocities. When $t_{e}/t_{k}$ is not too small, the transport rate is computed explicitly. When $t_{e}/t_{k} << 1$, this procedure is subject to numerical instabilities (the net turbulent flux is the small difference between large quantities). To avoid this, when $t_{e}/t_{k} < 0.1$, the dust/gas mixing ratio in that size range is simply assumed constant. In practice, this means aggregates must reach sizes $> 1$ cm before they begin to concentrate toward the central plane.

Results: An earlier publication (9) reported rapid coagulation, with a corresponding drop in opacity. The nebula became optically thin in $\sim 10$ yr, much shorter than the viscous evolution time. That result was erroneous, due to the use of the Völkl et al. parameterization for relative velocities of small grains. The revised algorithm discussed above for $t_{e} < t_{s}$ gives a much lower coagulation rate (thermal and differential settling velocities are also low) and slower evolution. For pure coagulation (no collisional breakup), optical depth remains high ($\sim 100$) for $> 10^{4}$ yr before beginning a gradual decline. Infall from the protosolar cloud may renew the grain supply and extend this phase. At $1.5 \times 10^{4}$ yr, the largest aggregates are $\sim$ tens of cm diameter, but represent a negligible fraction of the mass of solids. Most of the mass is in aggregates $\sim 10 \mu m$. The degree of settling in the central plane is negligible. These processes are slower if weak aggregates allow collisional breakup. These results are consistent with an extended period of viscous evolution in the turbulent accretion disk. Formation of planetesimals remains a problem; it is not yet clear that it can be accomplished within a reasonable disk lifetime.