COMPACTION OF ICY SATELLITES, Janusz Eluszkiewicz, Dept. of Earth Sciences, Parks Road, Oxford OX1 3PR, U. K. (on leave from: Institute of Geophysics, Warsaw University, Pasteura 7, 02-093 Warszawa, Poland)

The goal of this study is to calculate the extent of global contraction during a satellite's evolution and the present porosity distribution in its interior. As an aside, it is hoped that the silicate mass fraction C, defined as

$$C = \text{total mass of rock/total mass of the satellite} \tag{1}$$

would be evaluated.

The satellites of interest are medium-size icy satellites of giant planets, e.g. Mimas. These are the most suitable objects to be considered under the assumptions of the present model. A consideration of larger objects, e.g. the Galilean satellites of Jupiter, would involve additional complicating factors (such as phase transitions in ice, long lasting convection, melting, and differentiation) which are excluded here.

It is assumed that the satellite forms instantaneously as a spherical body composed of a homogeneous mixture of H_2O-icy and rocky grains (current models of the origin of satellites predict for them a formation time several orders of magnitude shorter than the age of the solar system). During formation, the assemblage of grains is compacted by rearrangement (preserving homogeneous mixture of ice and rock) under the weight of overlying layers. This process is described by an equation relating porosity ϕ to pressure p

$$\phi = \phi(p) \quad . \tag{2}$$

In equation (2) pressure is calculated assuming hydrostatic equilibrium [1].

Immediately after initial compaction the satellite has a radius R_o greater than its present value R* (for Mimas R* = 198.8 km). In general, R_o is a function of C and the parameters entering equation (2) (the total mass of the satellite is known and remains constant during evolution). If these parameters could be fixed, then effectively we would have

$$R_o = R_o(C) \quad . \tag{3}$$

Now, time-dependent compaction begins. This process can be described as a two-phase flow [2], i.e. pore space (assumed to be empty) and solid matrix. The basic equations are derived from laws of conservation of mass, momentum, and energy. In the present context they read

$$\partial\phi/\partial t = \nabla(1-\phi)\vec{v} \tag{4a}$$

$$\partial\{(1-\phi)\sigma^s_{ij}\}/\partial x_j - (1-\phi)\rho_s g_i = 0 \tag{4b}$$

$$(1-\phi)\rho_s C^s_p\{\partial T/\partial t\} + (1-\phi)\rho_s C^s_p\vec{v}\nabla T = \nabla k_T \nabla T + Q \quad , \tag{4c}$$

where the meaning of the symbols used is as follows:

$\vec{v}$: velocity of the compacting matrix

σ^s_{ij}: stress tensor acting on the solid matrix

ρ_s : density of the solid matrix

$\vec{g}$: acceleration due to gravity

x_j : Cartesian coordinates

C_p^s : specific heat of the solid matrix

T : temperature

k_T : effective thermal conductivity of the porous matrix

Q : source term owing to dissipation and the energy produced by the decay of radioactive elements contained in the rocky part of the matrix.

In order to close the set of equations (4), a relation between stress and strain rate and/or strain is required

$$\sigma_{ij}^s = \sigma_{ij}^s(\dot{\varepsilon}_{kl}, \varepsilon_{kl}) \quad . \tag{5}$$

To the best of my knowledge, relation (5) for granular ice or ice-rock mixtures at low temperatures has never been established experimentally. However, a detailed form of equation (5) has been derived theoretically and is now avaiable in a software package [3]. This allows equations (4) to be integrated over the age of the solar system, starting with the porosity distribution resulting from the initial compaction, equation (2), the initial value of the radius from equation (3), and other initial and boundary conditions obtained by standart means of planetary modelling. As a result, the value R for the present radius of the satellite would be obtained. If $R \neq R^*$, then the calculations should be repeated for another value of C.

A special solution of equation (4b) can be obtained by assuming Newtonian behaviour

$$(1-\phi)\sigma_{ij}^s = \zeta\{\partial v_k/\partial x_k\}\delta_{ij} + \eta(\partial v_i/\partial x_j + \partial v_j/\partial x_i - \tfrac{2}{3}\delta_{ij}\{\partial v_k/\partial x_k\}) \quad , \tag{6}$$

where ζ and η, respectively, are the bulk and shear viscosity of the porous matrix. In that case (and assuming spherical symmetry), equation (4b) may be solved analytically for the case of uniform porosity $\phi \equiv \phi_o$, giving

$$v = 2\pi G\rho_s^2(1-\phi_o)^2 r(r^2 - 3R^2)/\{15(\zeta+\tfrac{4}{3}\eta)\} \ , \tag{7}$$

where r is the radial distance from the centre of the satellite, R is the current radius of the satellite, m is the mass contained in the sphere of radius r, and G is the gravitational constant. This solution allows an estimate /via equation (4a)/ of the compaction time scale τ_o

$$-\frac{1}{\phi}\{\partial\phi/\partial t\}|_{r=0} \equiv 1/\tau_o = 2\pi G\rho_s^2(1-\phi_o)^3R^2/\{5\phi_o(\zeta+\tfrac{4}{3}\eta)\} \quad . \tag{8}$$

REFERENCES:

[1] Eluszkiewicz J. and Leliwa-Kopystynski J. (1987). To appear in the Proceedings of the 18th Lunar and Planetary Science Conference.
[2] McKenzie D. P. (1984). J. Petrology, 25, 713-765.
[3] Ashby M. F. (1987). Personal communication.