PLANETESIMAL COLLISION RATES: TRANSITION FROM RANDOM TO KEPLERIAN APPROACHES

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Simulations of early collisional accretion among planetesimals show incipient runaway growth of some larger bodies ("planetary embryos") as the remaining presence of many small bodies keeps random velocities in the system quite low (1,2). The low velocities help promote rapid growth of the embryos, because they give large gravitational cross-sections for collisions.

As long as encounters among planetesimals and embryos are governed by random velocities (orbital eccentricities and inclinations), dynamics of approach and encounter can be treated with a two-body approximation for purposes of estimating collision frequencies and gravitational acceleration. However, once an embryo gets so large that it can collisionally feed on planetesimals that have significantly different semi-major axes ($\Delta a > ea$), approaches are governed primarily by the keplerian shear of the system (bodies closer to the sun go faster than farther ones); the random motion becomes negligible. In this regime, adequate representation of the actual approach trajectory requires consideration of the three-body (sun-embryo-planetesimal) problem. Greenberg et al. (1) noted onset of this transition in their simulations, and for this reason could not extend them beyond the growth of embryos 500-1000 km in diameter. Here we describe a way to estimate the impact rate analytically for the keplerian regime.

First, assuming circular orbits, we estimate the maximum orbital distance Δa of a planetesimal on a circular orbit that can be perturbed during a synodic passage onto an embryo-crossing path. We equate the perturbing force $\sim Gm/(\Delta a)^2$ times the duration of the synodic passage $\sim 4/n$ with the velocity change required for a crossing orbit $\sim n\Delta a$, which yields $\Delta a \sim (4m/M)^{1/3}a$, or about 2 Hill radii. (See notation list below). This range roughly defines the feeding annulus of the embryo. Keplerian shear will dominate over random motion if $\Delta a \gtrsim ea$, which is equivalent to $V/V_e \lesssim 0.1$.

Next we estimate the rate of impact with the embryo for planetesimals approaching on nearly circular orbits within the feeding annulus for a two dimensional swarm. The synodic feeding flow (mass/time) is the product of the surface density σ, the flow rate $\sim n\Delta a$, and the annulus width $2\Delta a$. This material enters the embryo's Hill sphere near a Lagrange point with a velocity V_s relative to the embryo: $V_s \sim n\Delta a$ according to a Jacobi-constant analysis of the restricted three-body problem. Inside the sphere of influence, behavior can be approximated as two-body motion, so the impact probability is roughly the two-body gravitational cross-section $2rV_e/V_s$, divided by Δa. Combining the feeding flow with the impact probability gives

$$\text{Impact rate (2-D, keplerian)} \sim 4r\sigma V_e, \qquad \text{(A)}$$

similar to the formula for the random-motion regime:

$$\text{Impact rate (2-D, random)} \sim 2r\sigma V_e. \qquad \text{(B)}$$

The same method allows an estimate for the three-dimensional case. Here the synodic feeding flow is the same as in the 2-D case. The impact probability once inside the sphere of influence is the two-body gravitational cross-section, now $\pi r^2(V_e/V_s)^2$, divided by $\Delta a(2V/n)$, where $2V/n$ is the vertical thickness of the swarm. Thus, for the three-dimensional case with approach dominated by keplerian shear

$$\text{Impact rate (3-D, keplerian)} \sim \pi r^2 V_e{}^2 \sigma (4m/M)^{1/3}/(Va). \qquad \text{(C)}$$

Compare (C) with the formula that applies where random motion dominates:

$$\text{Impact rate (3-D, random)} \sim \pi r^2 V_e{}^2 \sigma n/(2V^2). \qquad \text{(D)}$$

These analytically derived impact rates are in good agreement with results of Monte Carlo experiments (3,4). The transition from random motion to keplerian domination at $V/V_e \sim 0.1$ is in excellent agreement with the break-down of Eqs. (B) and (D) found by Wetherill and Cox (4) near that value of V/V_e. In the 2-D case, they found "enhancement" of impact rates by a factor of 2 or 3 above what the two-body encounter formula would give, in accord with the ratio of our Eq. (A) to (B). Our formulae give "enhancement" in the 3-D case by the ratio of (C) to (D):

$$\text{"Enhancement"} \sim 2(V/V_e)(\rho/M)^{1/6}a^{1/2} \qquad \text{(C)/(D)}$$

At 1 AU this value becomes $30(V/V_e)$, which predicts a jump in impact rates by a factor 3 at the transition $V/V_e \sim 0.1$, and which gives a less steep dependence of impact rate on V/V_e for larger V/V_e than for smaller V/V_e, in excellent agreement with (4). Note also the increase with a (c.f. 5).

Our analysis as well as the experiments in (4) demonstrate the transition from the random regime to the keplerian regime. In (4) this distinction is not emphasized: Results are related primarily to V/V_e. That description is potentially misleading because it may suggest that the transition is due to break-down of the two-body approximation at low encounter velocities. In fact, as we have shown, there is an <u>a priori</u> qualitative distinction between the encounter modes in the two regimes. Moreover, even for small V in the keplerian regime, the actual encounter velocity is not small and the two-body approximation seems to work well if the appropriate encounter velocity (V_s) is used.

In planetary accretion models, the transition from the random to the keplerian regime can be partially modelled by increasing Eq. (D) by an "enhancement" factor. But other celestial mechanical differences must be considered. For example, if a keplerian encounter does not yield an impact, a planetesimal's post-encounter orbit must be farther from the embryo than its initial orbit, as in ring shepherding. Such radial removal from the feeding annulus could significantly modify growth rates.

<u>Notation</u>: a = semi-major axis; e = eccentricity; M = solar mass; m = embryo's mass; r = embryo radius; ρ = embryo density; n = mean motion; G = gravity constant; V = random or thermal velocity; V_e = embryo's surface escape velocity; V_s = velocity entering sphere of influence.

<u>References</u>: (1) Greenberg, R., et al. 1978, <u>Icarus</u> **35**, 1; (2) Wetherill, G.W., and Stewart, G.R. 1987, <u>LPSC</u> XVIII, 1077; (3) Nishida, S. 1983, <u>Prog. Theor. Phys.</u> **70**, 93; (4) Wetherill, G.W., and Cox, L.P. 1985, <u>Icarus</u> **63**, 290; (5) Lissauer, J., and Greensweig, Y. 1987, <u>LPSC</u> XVIII, 556.