Scaling Problems in Catastrophic Collisions

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Scaling laws have been developed for catastrophic collisions in order to better understand how small scale fragmentation experiments can be applied to asteroidal collisions (1,2). The form of the scaling laws depends on assumptions regarding the mechanical properties of the target body and the impactor. In particular, two models have been proposed which differ dramatically in their predictions of how collisional outcomes should depend on the size of the target. Unfortunately, the variations in target size required to discriminate between these models is probably larger than can be accommodated in the lab. The purpose of this abstract is to compare fragment velocities measured in the lab to the relative velocities observed for asteroid families (3) within the context of the two scaling models.

Before proceeding we define the following variables: E, the kinetic energy of the impactor; MT, the mass of the target body; D the diameter of the target; M1, the mass of the largest fragment produced in the collision; and Q=E/MT, the specific energy for the collision. A fragmentation threshold is defined in the usual manner such that the ratio M1/MT is a constant value. A characteristic velocity, v*, is defined to be the velocity such that, at the threshold, a specified mass fraction of the fragments are ejected faster than v*.

The first of the two scaling models is one that is traditionally used in discussions of collisional fragmentation. At the heart of this model is the assumption that, in the strength regime, fragmentation occurs whenever the specific energy, Q, exceeds a critical value, which is independent of D. From scaling arguments this model can be shown to hold only when the following two conditions are met: (a) the impactor is completely described by its kinetic energy (rather than energy and momentum) and (b) the mechanical properties of the target are independent of both size scale and loading rate (1,2). Under these conditions, v* can be shown to be independent of D in the strength regime. In the gravity regime, v* should increase linearly with D, due to self-gravitational effects.

The second scaling model discussed here is more general in its assumptions regarding the properties of the impactor and the target. The impactor is described by both its energy and momentum through a point-source coupling parameter (5). More importantly, the target material is assumed to have a strain-rate dependent fracture strength. This second assumption is motivated by a large number of laboratory measurements of dynamic fracture strength (e.g. 6, 7). Typically, strength is found to be proportional to the 1/4 power of the strain rate. Under these conditions one can show that the characteristic fragment velocity should be proportional to D\(^{1/7}\) in the strength regime and to D in the gravity regime.

Hence, the two scaling models give significantly different predictions in the strength regime. As a test we have compared the proposed scaling laws with laboratory collision experiments (8) and asteroid observations (see Figure 1). Zappala et al. calculated the mean relative velocities for families from the observed differences in the semimajor axes of the members. For each family they calculated the diameter of the original parent object from the total observed mass and an estimate of the "missing" (i.e. unobserved) mass. They also calculated a reliability index based on the relative magnitudes of the missing and observed masses. We chose to use only the families for which the reliability index was \(< 1.5\), which means the estimated unobserved mass should be no more than 50% of the total observed mass.

In order to compare velocities at the fragmentation threshold, we selected only the families for which 0.4 < M1/MT < 0.6, i.e. an interval centered on 0.5, which is the traditional definition of the fragmentation threshold. A small correction was applied to account for the fact that the actual value of M1/MT differed somewhat from 0.5. The approximate relations M1/MT = 1/Q and \(v = Q^{0.76}\) (e.g. refs 1, 8), which imply \(v = (M_1/MT)^{-0.76}\), were used to correct each family to the desired mass ratio of 0.5. In all cases the corrections were small, shifting the points at most one or two times the width of the data symbols shown in the figure.

The data point from Fujiwara and Tsukamoto (ref. 8) represents a 2.8 km/sec impact on a 10 cm diameter basalt sphere. The event, with value M1/MT=0.34, is near the fragmentation threshold. The observed fragment velocity distribution implies a median ejection velocity of only a few to several meters/sec.
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The two scaling models discussed above are also shown in Figure 1. In the strength regime, the constant fracture stress model requires that the characteristic velocity is constant while the strain-rate model implies that the velocity decreases as $D$ increases. In both cases, the velocity will increase with $D$ when in the gravity regime, which is represented by a line of slope 1 drawn through the asteroid data. Unfortunately, the asteroid data do not extend to small sizes ($D=1-10$ km), where the two models exhibit their greatest differences.

It is interesting to note that both models imply that the transition from the strength regime to the gravity regime occurs at a rather small diameter. Numerical simulations of the evolution of the asteroid size distribution (9) require that the strength/gravity transition occur at a diameter of around 100-150 km, in order to reproduce the features observed in the current population. This is an order of magnitude larger than the transition value suggested by the constant fracture stress model in Figure 1. The strain-rate model gives an even larger discrepancy. If one were to adopt a transition at $D=100$ km (by shifting the gravity line to the right) then the predicted velocities in the gravity regime would be much smaller than observed for the families, and in fact much lower than the escape velocity of the parent body.

We are therefore left with two problems. First, how can one reconcile the strength/gravity transition diameter inferred from the velocity observations with the values required by collisional evolution models? Second, the strain-rate model provides a bigger discrepancy than does the constant fracture stress model. Yet there is a great deal of experimental evidence to suggest that fracture of rock is strain-rate dependent. This has been demonstrated not only by laboratory measurement but also by observations of high explosive cratering events in the field. These problems are currently being addressed through further scaling analyses.