ANALYSIS OF ZODIACAL LIGHT DATA BASED ON A FINITE HOMOGENEOUS DUST CLOUD.

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In interpretations of zodiacal light data it is mostly assumed that the zodiacal dust cloud is of infinite extent. Since Pioneer data have revealed, however, that hardly any sunlight is scattered by dust particles beyond about 3 AU, this assumption may be wrong ([1],[2],[3]). Therefore, we have developed a theory for analysing zodiacal light data on the basis of a finite dust cloud having scattering properties not depending on the distance, r, to the Sun [4]. We assume a particle number density in the cloud of the form

$$n(r) = \begin{cases} n_0(r_0/r)^{\nu} & \text{if } r \le r_m \\ 0 & \text{if } r > r_m, \end{cases} \tag{1}$$

where r_0 is the distance from the Earth to the Sun, n_0 the number density at $r=r_0$ and r_m is the value of r at the boundary of the cloud. The brightness of the zodiacal light seen by an earthbound observer looking in the ecliptic plane at an elongation, ϵ, may then be written as

$$I(\epsilon) = \frac{F_0 \cdot n_0 \cdot r_0}{(\sin \epsilon)^{\nu+1}} \int_{\epsilon}^{\theta_m} (\sin \theta)^{\nu} \, [\bar{s}\, \varphi(\theta)]_{r_m} \, d\theta. \tag{2}$$

Here, F_0 is the solar flux at r_0, θ is the scattering angle, $\bar{s}$ the mean total scattering cross section and $\varphi(\theta)$ the mean volume scattering function.
The upper boundary, θ_m, in the integral is the maximum scattering angle occurring at the point of intersection of the line of sight and the boundary of the cloud; θ_m is given by

$$\sin \theta_m = \frac{r_0}{r_m} \sin \epsilon. \tag{3}$$

Further, the normalization of $\varphi(\theta)$ is so that

$$\int_{4\pi} \varphi(\theta) \, d\omega = 1, \tag{4}$$

where $d\omega$ is an element of solid angle. It should be noted that we have added a subscript r_m in $[\bar{s}\, \varphi(\theta)]_{r_m}$. This is done because for one and the same brightness distribution, $I(\epsilon)$, the scattering properties of the cloud cannot be the same for different r_m- values.

Observing the fact that not only $I(\epsilon)$ but also its rate of change with elongation does not vary with r_m we devised an algorithm for computing

$$[\bar{s}\, \varphi(\theta)]_{r_m} \,/\, [\bar{s}\, \varphi(\theta)]_{\infty}. \tag{5}$$

Some results are displayed in Fig. 1 for r_m=3 AU and 3.5 AU. For the mean volume scattering function in $[\bar{s}\, \varphi(\theta)]_{\infty}$ we have used the function employed in [5] on interpreting the observed brightness in terms of an infinite cloud having a particle density $n(r) \propto r^{-1.3}$. It is seen from Fig. 1 that differences up to about 10% may occur between $[\bar{s}\, \varphi(\theta)]_{r_m}$ and $[\bar{s}\, \varphi(\theta)]_{\infty}$ for the values of θ and r_m considered. Similar calculations indicate that the polarization properties of the particles, as deduced from earthbound observations, hardly change with varying r_m.

We have also studied the effects of finiteness on the brightness observed from a space vehicle at a solar distance, R, in the ecliptic plane. In a homogeneous infinite cloud with a particle density distribution given by $n(r) \propto r^{-\nu}$ one would expect for the observed brightness

$$I(\epsilon,R) \propto R^{-(\nu+1)}. \tag{6}$$

To calculate the brightness in a finite cloud we considered dust particles with scattering properties providing a good explanation for the earthbound observations. In particular, we used ν=1 and the linear combination of three Henyey-Greenstein functions given in [5] for the phase function. Some results are shown in Fig. 2, where $I(\epsilon,R)$ is measured in the usual $S_{10}(V)_{\odot}$ units. Evidently, Eq. (6) is not fulfilled in these cases; in particular, large deviations occur for values of R near the edge of the cloud.

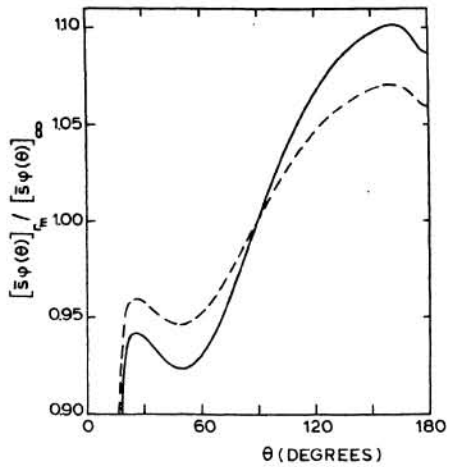

Fig. 1 The ratio of the mean differential scattering cross sections of a finite and an infinite cloud as a function of scattering angle, θ, for r_m=3 AU (solid curve) and r_m=3.5 AU (dashed curve).

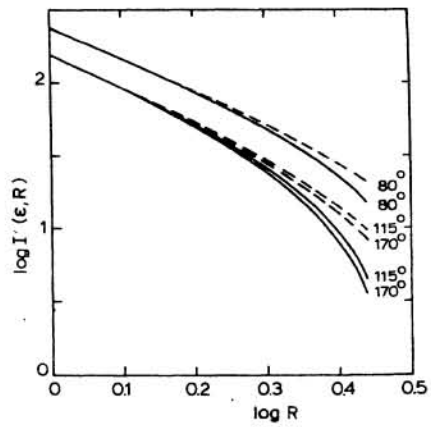

Fig. 2 The brightness of the zodiacal light as a function of solar distance for ε=80°, 115° and 170°. Observations are assumed to be made by a spacecraft in a finite dustcloud. Solid lines represent the calculated brightness for r_m=3 AU, dashed curves show the result for r_m=3.5 AU.

We can make a comparison with the analysis of Pioneer data of the zodiacal light by Schuerman [6] who reported considerable deviations from Eq. (6). Attempting to fit a power law to the observed brightness he found, for instance, that $I(\epsilon,R) \propto R^{-3.5 \pm 1.4}$ at ε=115° and for 1.35 AU < R < 2.64 AU. The exponent of R is pretty far off from what might be expected for an infinite cloud since it is usually assumed that ν+1 is about 2.3. In addition, the uncertainty (± 1.4) is quite large. From a linear least squares fit for the same range of R-values we found that our values of I(ε,R) for r_m=3 AU and ε=115° [see Fig. 2] can be represented by a power law according to $R^{-3.5}$.

It is clear that the finiteness of the dust cloud is an important fact to consider in discussing deviations from a power law behaviour of the brightness observed from a spacecraft. Our tentative treatment does not suffice to conclude that the finiteness alone may explain its dependence on R. Inhomogeneities may also play a role in this respect. Future analysis of data from deep space probes should nevertheless seriously consider the possibility of finite cloud dimensions.

Acknowledgement

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