TWO-DIMENSIONAL FRAGMENTATION HYDROCODE

Collisions between solid bodies, a dominant process in the solar system, have been studied both experimentally (Fujiwara et al., 1989) and theoretically (Grady and Kipp, 1980). We report on the development of a two-dimensional numerical hydrocode, modified to include material strength effects, which will be used to model high-velocity collisions. The output from this code includes a complete fragmentation summary for each cell of the modeled object: fragment size distributions, vector velocities of particles, peak values of pressure and tensile stress, and peak strain rates associated with fragmentation. Contour plots showing pressure and temperature at given times within the object are also produced. By invoking axial symmetry, three dimensional events can be modeled, such as zero impact parameter collisions between asteroids.

Grady and Kipp's (1980) equations for fragmentation resulting from tensile stress in one dimension have been implemented in a numerical routine based on Los Alamos's two-dimensional hydrocode 2-D SALE (Amsden et al., 1980). Grady and Kipp originally used volume strain in place of linear strain when extrapolating to higher dimensions. But because volume strain can remain zero even when tensile strain causes fragmentation to occur along one axis, this led to inaccurate results. We have instead decomposed the stress tensor into its principal components, treating the stress along the most tensile component as a one-dimensional problem.

We use the hydrocode in its fully explicit, Lagrangian mode, and have assumed a uniform material throughout the object. In order to directly compare our model with the one-dimensional numerical results of Melosh (1987), we use a linear Murnaghan equation of state, although the more sophisticated Tillotson constitutive relation (Melosh, 1989) with a Hugoniot elastic yield stress has also been used.

The Grady-Kipp fragmentation model assumes an original Weibull distribution of crack nucleation centers

\[ n(\varepsilon) = ke^{-m}\varepsilon \]

where \( n \) is the number of flaws that are activated at or below a tensile strain \( \varepsilon \). Constants \( k \) and \( m \) are material properties. As flaws are activated the elastic moduli are decreased by a factor \((1-D)\), where \( D(t) \) represents the damage due to crack growth. When \( D=1 \) the material is completely fractured and can support no tensile stresses. We use the same approximation to the original Grady and Kipp equation used by Melosh (1987), namely,

\[ \frac{dn^{1/3}}{dt} = \frac{8\varepsilon}{3a^{1/3}m/3} \]

where \( a \) is defined in terms of the Weibull parameters \( k \) and \( m \) and the crack velocity \( c_g \):

\[ a = \frac{8\varepsilon K}{(m+1)(m+2)(m+3)} \]
The fracture area $A(t)$, from which fragment size is derived, is then given by

$$\frac{dA}{dt} = \frac{(m+2)(m+3)}{20g} \varepsilon^{2m/3} D^{1/3} a^{2/3}$$

which is exact for constant strain rate. The fragment diameter at the maximum of the fragment size distribution, $L_{\text{GK}}$, can be shown to be

$$L_{\text{GK}} = \frac{3(m+3)}{(m+2)} \frac{1}{A(t_f)}$$

where $t_f$ is the time at which the damage $D(t)$ reaches 1.

For the purpose of testing our code, we injected a uniform compressive pulse into the bottom of a 300m x 1m rectangle of material (anorthosite). The pulse rises linearly to a peak particle velocity (100 to 1000 m/s in our runs) over a distance of 50m, and decays linearly to zero particle velocity over a distance of 200m. Such a pulse corresponds to the Grady-Kipp model's implicit assumption of a uniform strain rate. As the pulse propagates towards the top of the block, it reflects at the free surface and becomes tensile. It is this tensile stress which fragments the material.

Figure 1 demonstrates the accuracy of the two-dimensional integration in comparison with the one-dimensional results of Melosh (1987) and the analytical solutions of Grady and Kipp (1980) for a linearly increasing tensile stress (constant strain rate). The 2-D model shows no systematic deviation from the analytical solutions, and shows a closer correlation than the 1-D results.

The development of Grady-Kipp fragmentation theory within the framework of a 2-D hydrocode has many advantages over analytical approaches, and promises to be of considerable use in modeling asteroid collisions, impact cratering, disruption of co-orbital satellites, and other important processes in the solar system.