OBLIQUITY HISTORIES OF EARTH AND MARS: INFLUENCE OF INERTIAL AND DISSIPATIVE CORE-MANTLE COUPLING; Bruce G. Bills, LPI, Houston, TX

For both the Earth and Mars, secular variations in the angular separation of the spin axis from the orbit normal are suspected of driving major climatic changes (1,2,3,4). There is thus considerable interest in determining, as accurately as possible, the amplitude and timing of these obliquity variations. If the orientation of the orbital plane were inertially fixed, and the planet were to act as a rigid body in its response to precessional torques, the spin axis would simply precess around the orbit normal at a fixed obliquity ϵ and at a uniform angular rate $\alpha \cos(\epsilon)$. The precession rate parameter

$$\alpha = \frac{3}{2}\frac{(C-(A+B)/2)}{C\omega} \quad \Sigma\, \mu_i/(b_i)^3 \qquad [1]$$

depends on the principal moments of inertia ($A \leq B \leq C$) and rotation rate ω of the perturbed body, and on the gravitational masses $\mu = GM$ and semiminor axes $b = a(1-e^2)^{1/2}$ of the perturbing bodies. For Mars, the precession rate is not well known, but probably lies in the interval 8-10 arcsec/year (5,6,7,8,9,10). The much larger precession rate for the Earth (~50.3 arcsec/year) is partly due to greater proximity to the Sun (which contributes roughly 1/3 of the total), but is mostly due to the presence of the Moon (11).

Gravitational interactions between the planets lead to secular motions of the orbit planes of the form

$$\sin(I)\, e^{i\Omega} = \Sigma\, N_j e^{i(s_j t + \delta_j)} \qquad [2]$$

where I and Ω are the inclination and longitude of the node, and N_j, s_j and δ_j are amplitudes, rates and phase constants (12,13). In the rigid body case, the spin axis still attempts to precess about the instantaneous orbit normal, but now the obliquity varies. A first order solution for the obliquity can be written in a form similar to [2] but with amplitudes $K_j N_j$, where the admittance K_j has the value (14,15,16)

$$K_j = \frac{s_j}{s_j + \alpha \cos(\epsilon)} \qquad [3]$$

As the orbital precession rate constants s_j are all negative and fall within the range {-26.3 arcsec/year $\leq s_j \leq 0$}, while the spin precession rate constants are positive, the potential exists for significant resonant amplification of the obliquity if the denominator of [3] approaches zero (17,18). The possibility of actual singularities in [3] is not too worrisome, as the linear analysis leading to that form of the admittance is no longer applicable in the immediate vicinity of a resonance. However, the physical model of rigid rotation which leads to that formula is almost certainly too simplistic.

The hydrostatic figure of a planet represents a compromise between gravitation, which attempts to attain spherical symmetry, and rotation, which prefers cylindrical symmetry (19). Due to their higher mean densities, the cores of the Earth and Mars will be more nearly spherical than the outer layers of these planets. The direct gravitational torques on the core will thus be inadequate to make it precess at the same rate as the mantle. For the Earth, where the structure is relatively well known, the core oblateness is only about 3/4 that required for coprecession with the mantle (20). However, it is clearly the case that the core and mantle precess at very nearly the same rate (21,22). Two different types of torques contribute to the coupling.

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On short time scales it is appropriate to consider the core to be an invisid fluid constrained to move within the ellipsoidal region bounded by the rigid mantle (23,24). The inertial coupling provided by this mechanism is effective whenever the ellipticity of the container exceeds the ratio of precessional to rotational rates. If the mantle were actually rigid, or even elastic, this would be an extremely effective type of coupling. However, on sufficiently long time scales, the mantle will deform viscously and can accomodate the motions of the core fluid. The inertial coupling torque exerted by the core on the mantle will have the form $T_i = k_i [\omega_m \times \omega_c]$. A fundamentally different type of coupling is provided by electromagnetic or viscous torques (25,26). The dissipative coupling torque exerted by the core on the mantle will have the form $T_d = -k_d [\omega_m - \omega_c]$. This type of coupling is likely to be most important on longer time scales. In each case, the mantle exerts an equal and opposite torque on the core.

The admittance which relates inclination amplitude to obliquity amplitude is now a complex quantity which can be written in the following form (27,28)

$$K(s) = \frac{s(s + a_m + \eta)}{(s + a_m)(s + a_m + \eta) - \Delta a\, \eta_m} \qquad [4]$$

The coupling constants are rescaled ($\beta_m = k_d/C_m$, $\gamma_m = \omega \cos(\epsilon)\, k_i/C_m$) and then combined to form a single complex parameter $\eta_m = \gamma_m - i\beta_m$. Also, $a_m = \alpha_m \cos(\epsilon) + \cos(I)\, d\Omega/dt$. Core parameters ($a_c,\beta_c,\gamma_c,\eta_c$) are defined analogously, and the sum of core and mantle parameters $\eta = \eta_m + \eta_c$ is left unsubscripted, and $\Delta a = a_m - a_c$. Viscous relaxation of the mantle is included by multiplying both γ values by $(is\tau)/(1+is\tau)$, where τ is the effective Maxwell relaxation time of the mantle.

There are several features to note, in comparing [3] and [4]. As is typical of a forced oscillation with damping, the response now lags the forcing by an amount which depends on the frequency of forcing and the strength of the viscous coupling. The most evident difference in response is near resonance, since [4] exhibits no singularities. However, even away from resonance, the inclusion of possible differential precession can modify the obliquity history by amounts that could have climatic significance. Unfortunately, the coupling constants are not well known, even for the Earth, and are almost completely unconstrained for Mars. Thus, some caution is advised in constructing climatic history scenarios which depend on details of the obliquity history.

References: (1) M. Milankovitch, Konin. Serbische Akad., 484p, 1941; (2) A.L. Berger et al. (eds.) Milankovitch and Climate, D. Reidel, 1984; (3) O.B. Toon et al., Icarus 44, 552-607, 1982; (4) S.M. Clifford et al., Eos 47, 1585-1596, 1988; (5) P. Lowell, Astron. J. 28, 169-171, 1914; (6) W.M. Kaula, Geophys. Res. Lett. 6, 194-196, 1979; (7) B.G. Bills, Geophys. Res. Lett. 16, 385-388, 1989; (8) B.G. Bills, Geophys. Res. Lett. 16, 1137-1338, 1989; (9) A.T. Sinclair, Astron. Astrophys. 220, 321-328, 1989; (10) T.A. Morley, Astron. Astrophys. (in press) (11) H. Kinoshita, Celest. Mech. 15, 277-326, 1977; (12) P. Bretagnon, Astron. Astrophys. 30, 141-154, 1974; (13) J. Laskar, Astron. Astrophys. 198, 341-362, 1988; (14) S.G. Sharaf, & N.A. Budnikova, Trud. Inst. Teor. Astron. 11, 231-261, 1967. (15) W.R. Ward, J. Geophys. Res. 1974; (16) A. Berger, Celest. Mech. 15, 53-74, 1977; (17) W.R. Ward et al., J. Geophys. Res. 83, 243-259, 1979; (18) W.R. Ward, Icarus 50, 444-448, 1982; (19) Z. Kopal, Figures of Equilibrium of Celestial Bodies, U. Wisc. Press, 1960; (20) M.L. Smith & F.A. Dahlen, Geophys. J. R. Astr. Soc. 64, 223-281, 1981; (21) E.C. Bullard, Proc. Roy. Soc. A197, 433-453, 1949; (22) F.D. Stacey, Geophys. J. R. Astr. Soc. 35, 47-55, 1973; (23) H. Poincare, Bull. Astr. 27, 321-356, 1910; (24) A. Toomre, Geophys. J. R. Astr. Soc. 38, 335-348, 1974; (25) S. Aoki, Astron. J. 74, 284-291, 1969; (26) M.G. Rochester, J. Geophys. Res. 67, 4833-4836, 1962. (27) P. Goldreich & S.J. Peale, Astron. J. 75, 273-284, 1970; (28) W.R. Ward & W.M. DeCampli, Astrophys. J. Lett. 230, 117-121, 1979.