Scaling issues form the cornerstone of the study of the phenomena arising from impact cratering. Most significant results require assumptions about how the processes are affected by impactor velocity, impactor size, gravity, physical properties, atmospheric composition, and other factors. The answers to many questions are profoundly affected by the choice of the scaling methodology; but, at the same time, fundamental questions regarding the basis and proper choice of that scaling methodology remain. The differences in predictions with existing rules when extrapolating from laboratory tests to events with kilometer-sized impactors can range over orders of magnitude.

Recent research [1,2,3] has determined that the basis for the form of many scaling laws is the existence of a single "coupling-parameter" scalar measure of the impactor size, composition and velocity, and that that single measure arises from and governs the point-source solutions that evolve soon after the initial impact. It is the existence of that single measure that requires many scaling laws to have the observed power-law forms. However, the physical bases for the value of the governing exponents of those power laws is not known. An example is the observed dependence of impact crater volume on the $\pi^2$ gravity scaled size parameter [4,5] with a power of about -0.5 for dry sand, as observed in laboratory experiments. It is not known for certainty whether that same scaling should apply to mega-sized events, or to impact velocities of, say, 30 km/sec since the experiments cannot directly simulate those conditions.

To clarify, it can be shown that any point source measure, i.e. any coupling parameter, must have the power-law form $a U^\mu$ in terms of the impactor radius $a$ and the velocity $U$, where $\mu$ is an exponent with values that can be shown theoretically to be between $1/3$ to $2/3$ and seem to be about 0.4 for dry soils and about 0.6 for rocks and water. Once that exponent is known, a multitude of scaling results (e.g. cratering efficiency with event size, shock pressure decay with distance, ejecta distributions, and so on) can be derived, in terms of the exponent $\mu$ (see [3]). However, the value of that exponent must be found by other means. One way is to use experimental results for one phenomena (for example the dependence of the crater volume on impactor velocity or size), to determine $\mu$, and the use that value to predict other results that also depend on $\mu$. (The fact that those cratering results are indeed power-law is a strong experimental verification that they are in fact governed by a point-source coupling parameter). That approach is the basis of a number of the publications of my colleagues and I.

An alternative approach is to attempt to determine the coupling parameter (and perhaps the entire solution) directly from a theoretical formulation of the problem, i.e. from the equations of the balance of mass, energy, momentum, and the equations that describe the material properties of the media in question. This reduces to a search for point-source solutions. That has been done in special cases, the best known example is the solution for a spherical blast wave in a perfect gas that was determined in the 1940's by G.I. Taylor and others, and is often used in nuclear airblast analyses. In that case, it turns out that it is in fact the energy of the source that is the coupling parameter. Due to the fact that this is also the only well-known theoretical solution, many researchers seem to believe that the resulting "energy scaling" is the only possibility. The literature is full of papers where the "energy scaling" assumptions are made, either specifically or implicitly, apparently without the recognition of the special nature of that assumption. (For example all papers on cratering before the centrifuge results of the last decade.) A very different type of solution, and one of more interest to impacts in soils is one given in the Soviet literature in 1956 by Kompaneets for a "perfectly-porous" material [6]. In that solution, any non-zero porosity will give scaling that is not energy scaling, with results that can be substantially different. For that model, one can determine a specific algebraic relation for the coupling parameter (and the exponent $\mu$ above) in terms of the porosity of the material.
This background makes it clear that much could be learned about scaling issues for impacts into materials such as soils, rocks and ices if more general point source solutions could be determined. Unfortunately, for a general solid material, no such solutions have been previously determined, for several reasons. The solutions desired are two-dimensional. Even for the spherically-symmetric case, the equations of state for solids are much more complicated than those for a perfect gas, due to the presence of the cold compression component in the equation of state.

A number of cases have been found where analytical solutions exist, and will be presented. The solutions are based on various approximations of the equation of state over the range of the problem where the coupling of the source into the problem occurs. As an example, consider the impact at a few km per sec into a dry porous soil. The maximum pressures will be on the order of 100 kilobars or so. The adjacent figure shows the resulting equation of state as a plot of the Hugoniot curve of pressure versus density, in the case where the crush-up pressure is small compared to the 100-kilobar scale of the plot. The unloading adiabats are indistinguishable from the crush-up curve on the right. This model is well approximated to first order by the perfectly-porous model that assumes that, after crush, the material is incompressible, as modeled by the dashed line. However, it is also clear that this approximation becomes increasingly poor if the impact pressures were substantially greater than 100 kbar.

Even more significant and general are results for material models that approximate a general solid material with porosity, where the underlying, non-porous material model is a Mie-Gruniesen model with a linear $U_S-U_P$ Hugoniot relation. The resulting equation of state is depicted below. Results will be presented that give the scaling exponents as a specific function of the material parameters defining the porosity, the Gruniesen parameter, and the $U_S-U_P$ slope. Special limiting cases include the perfect gas, energy results of G.I. Taylor at one extreme, and the perfectly porous Kompaneet scaling at the other. The determination of these solutions and studies of the resulting scaling in specific applications give important and immediate results about the scaling of impact events in geophysical materials.

REFERENCES: