Thermal Models of Insolation-Driven Nitrogen Geysers on Triton: R. L. Kirk, Branch of Astrogeology, U.S.G.S., Flagstaff, AZ 86001

Introduction A highlight of the Voyager 2 encounter with Triton was the discovery of geysers-like plumes in the atmosphere, along with clouds and surface deposits (strands) that may be related to the plumes in origin [1]. At least two active plumes were observed at multiple emission angles, rising vertically to an approximate altitude of 8 km and flowing over the surface for several kilometers. These plumes were at 56° and 57°S latitude, respectively, well within the south polar cap and close to the current subsolar latitude of 45°S. The observed limb and terminator clouds were also close to the subsolar latitude, suggesting that the plumes are powered (or at least in some ways triggered) by insolation, though the presence of plume-like clouds on the terminator means that the eruption process is able to continue through the night. The rough theoretical ratio of the number of surface streaks to the number of plumes and clouds suggests a typical active lifetime for the plumes of roughly 5 years.

Photometric analysis (Soderblom, et al., work in progress) of the plumes and their shadows indicates that the plumes contain a large amount of low angular depth material on the order of 0.05. If (1) the dust grain size is \( \sim 1 \mu m \) (constrained from below by the need for significant optical absorption and from above by the lack of significant settling of the plume cloud along its length), and (2) the horizontal velocity is \( \sim 10 m s^{-1} \) (Pogson, submitted to Nature), the dust entrainment rate is \( \sim 20 \) times greater, a power of \( 5 \times 10^5 W \) is required to sustain the eruption. The energy expended over 5 years is then \( \sim 8 \times 10^{24} J \). Modeling of the geysers-like aspect of the plumes (Soderblom, et al., work in progress) indicates that venting of nitrogen with an initial altitude of 4 K above the ambient surface temperature (a region 100 m in diameter) can roughly account for the observed plume height of 8 km. This 100 m is the diameter of the base of the plume and probably corresponds to the size of a region, closely spaced vents rather than a single large orifice. The estimate is relatively "soft" in that the plume height is proportional to the square root of the geysers radius; furthermore, the actual height of eruption may be limited by atmospheric stratification rather than by the source radius and velocity of escape from the vents.

Smith et al. [1] sketched an insolation-driven model for the plumes in which heat is trapped by a greenhouse layer of clear nitrogen ice, which increases the subsurface temperature and creates a reservoir of high-pressure gas to feed the geysers in subjacent regions. Of this "reservoir" it is important that it is necessarily "charged" and "discharged" in temporally distinct episodes. The term "conduit" would perhaps be more appropriate.) Such a model is not ruled out at least by very simplistic estimates of the available energy. Deposition of the solar flux (1.5 W m\(^{-2}\) peak value, with a diurnal average roughly half this in the polar regions) at the base of a 2-nitrogen layer with a conductivity of \( \sim 0.24 W m^{-1}K^{-1} \) [2] could result in a subsurface temperature as high as 5 K above the ambient 37 K. A circular region as small as a 1.3 km radius would suffice to supply the energy contained in the reservoir, provided it could be channeled to the vent with perfect efficiency. My purpose outlined here is to investigate, using scaling arguments supported by numerical calculations, the extent by which energy can be transferred from an extended greenhouse absorber to a central geysers. Only when this energy transport process—in particular, its efficiency—is understood can we evaluate the plausibility of the insolation-driven geysers-based the dimension estimate for the "reservoir" for the plumes.

Energy-Transport Processes Two processes of energy transport will operate in a porous sub-greenhouse layer on Triton. The first is ordinary thermal conduction. The second results from the strong variation of the equilibrium vapor pressure of nitrogen with temperature. Localized injection of heat will result in increased pressure and hence gas flow through the pores toward cooler regions, where the gas will partly condense to maintain local equilibrium; because of the latent heat of vaporization, energy as well as mass will be transported. (I assume for simplicity that the amount of mass transport is insufficient either to change the porosity significantly or to exhaust the local supply of nitrogen, which may be a thin coating on a water-ice substrate rather than a pure nitrogen layer. These effects may in reality be important.) This second process is analogous to the operation of the vapor-filled "heat pipes" used, among other things, for thermal control in spacecraft. The importance of energy transport by the gas depends on the temperature (which determines the vapor pressure and density) and the D'Arcy permeability \( k_p \) of the region. If one ignores all but the exponential effects of temperature on the gas-flow rate when taking derivatives, one can conveniently define a "potential temperature" \( \Theta = T + L \frac{\partial P}{\partial T} \frac{T}{k_B} \) that appears in place of \( T \) in the thermal flux equation, in particular, obeys Poisson's equation in steady state. (There is also a negligible correction to the heat capacity that multiplies \( \frac{c}{\Theta} \).) In this equation \( L \) is the latent heat of sublimation, \( R \) the gas constant divided by molar mass, \( \rho \) the gas density, \( \eta \) the viscosity, and \( k \) the ordinary thermal conductivity. One can also calculate an "effective thermal conductivity" \( k_{eff} = k_0 \Theta / \Theta T \). The figure 1 shows \( k_{eff} \) as a function of temperature for nitrogen gas \( \rho = 0.1 \) and several different sizes, calculated using a simple porosity-permeability model [3], along with the ordinary conductivity of \( N_2 \) [2] and of \( H_2O \).

It is clear that we must appeal to highly efficient energy transport by the gas phase to make the insolation-driven geysers work. The reason is that the drops in temperature, both from the sub-greenhouse layer upward to the surface and horizontally toward the geysers, are comparable, and the horizontal distance is much larger. Thus, if the effective subsurface conductivity is not much larger than the conductivity of the greenhouselayer, the flux of energy toward the geysers will be very small. Numerical models confirm that the energy passed by ordinary conduction to a central geysers region by a surrounding lollipop operating as a greenhouse is nearly equal to the energy absorbed at the geysers region itself. A "leaky greenhouse" geysers with or without a surrounding collector of solar energy could operate at the inferred power, but it would have to be kilometers in diameter. I will therefore use a porosity of 0.1 and an effective grain size of 2 m in the following case—which is more or less the corresponding permeability is \( k_p \approx 10^{-4} m^2 \), far in excess of the values reported for fractured terrestrial lava [5], but only slightly beyond the range of values commonly tabulated for sediments [3]. Furthermore, a possible mechanism may exist on Triton for maintaining a nitrogen layer of the required porosity and coarseness. The equilbrium surface temperature of 37 K is close to the transition to the liquid (or "granular") phase. Poisson's equation in steady state is then also a negligible correction to the heat capacity that multiplies \( c/\Theta \). In this equation \( L \) is the latent heat of sublimation, \( R \) the gas constant divided by molar mass, \( \rho \) the gas density, \( \eta \) the viscosity, and \( k \) the ordinary thermal conductivity. One can also calculate an "effective thermal conductivity" \( k_{eff} = k_0 \Theta / \Theta T \). The figure 1 shows \( k_{eff} \) as a function of temperature for nitrogen gas \( \rho = 0.1 \) and several different sizes, calculated using a simple porosity-permeability model [3], along with the ordinary conductivity of \( N_2 \) [2] and of \( H_2O \).

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Steady-State Thermal Models Making a realistic model of a solar-driven geysers would be complicated. The model would depend on a number of extremely nonlinear (because the atmosphere and surface conditions) and nonlocal boundary conditions are also complex: (1) a mixed boundary condition on the "collector" region, where the nonlinear heat flux downward and the linear flux through the greenhouse layer must sum to the solar input, and (2) an even more involved temperature-energy relation where gas is extracted to run the geysers. A good deal can nevertheless be learned from much simpler models. I have calculated steady-state solutions to the energy transport equation with cylindrical symmetry to approximate appropriate boundary conditions. I approximate the "geysers" by a circular disk on the axis maintained at the ambient (potential) temperature; a real geysers can extract no more energy from the system than such a cold patch can without violating the second law of thermodynamics. On the "collector" (a larger annulus concentric with the geysers) I impose either Neumann (fixed heat...
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flux) or Dirichlet (fixed potential temperature for boundary conditions. These conditions, respectively, are the appropriate limits of a realistic boundary condition for small or large collectors. Unlike the true boundary condition, which involves both linear conduction to the boundary and nonlinear conduction, the linearized form involves only the nonlinear mechanism. Hence they allow simple scaling of a solution to the linear problem for the potential temperature to obtain the temperature field and fluxes for any given size of the real system. To achieve the steady-state solution, I used an explicit time-stepping scheme for the linear diffusion equation [7], with a multigrid scheme [6] to achieve convergence on a 513 by 513 grid in reasonable time, despite the small timesteps required for stability.

Figure 2 shows the potential temperature field for a Neumann boundary condition and no geyser; the peak potential temperature scales as the collector radius $R_{g}$: $\Delta \Theta_{\text{max}} \propto R_{g} / R_{d}$, where $R_{g}$ is the imposed flux. If the nonlinear conduction mechanism dominates (as assumed), the actual temperature then increases as the logarithm of $R_{g}$. Let $R_{g}$ be a relatively small fraction of the available solid reservoir $R$, so that $R_{g} / R_{d} \ll 1$, say $0.1 \text{ W m}^{-1} \text{ K}^{-1}$. The flux is being used to supply the greenhouse. The Neumann model then achieves a self-consistent central temperature of $41 \text{ K}$ for $R_{d} \approx 30 \text{ km}$. Smaller collectors would have lower central temperatures (though not as low as the Neumann model predicts, because the available heat flux would increase somewhat), while larger collectors would resemble the Dirichlet model and would not achieve temperatures much higher than $41 \text{ K}$.

For a Neumann model with a geyser of radius $R_{g} \ll R_{d}$, the output power obeys $P_{\text{out}} \propto f_{R_{g}} R_{g}$, because the geyser collects a flux of $\propto k \Delta \Theta_{\text{max}} / R_{d}$ over an area proportional to $R_{g}^2$, with $\Delta \Theta_{\text{max}}$ as given above. Requiring $P_{\text{out}} \approx 5 \times 10^{4} \text{ W}$ with $R_{d} = 30 \text{ km}$, we find that $R_{g} \approx 850 \text{ m}$. The size of the collector is not important above the limit given; a Dirichlet model with an infinitely large collector at a temperature of $41 \text{ K}$ yields the required $P_{\text{out}}$ for $R_{g} \approx 430 \text{ m}$. (The slight discrepancy is due to an artifact of the numerical model: imposing a sharp discontinuity in temperature between collector and geyser leads to a weak divergence of $P_{\text{out}}$ as the mesh is made finer.) These values of $R_{g}$ are still in excess of the target value of $50 \text{ m}$ estimated above, but they are an order of magnitude less than the $\geq 3.5 \text{ km}$ required for a "leaky greenhouse" with no surrounding collector. In these estimates, $R_{g}$ scales inversely with the assumed permeability $k_{d}$.

Discussion

The steady-state models discussed above cannot be taken literally, though I argue that they may be a reasonable guide to the scaling of the geyser mechanism. As Figure 2 shows, the steady-state thermal field is roughly hemispherical. The time required to heat the subsurface of Triton to a depth comparable to $R_{g}$ would, however, far exceed one Tritonian season. Forty Earth years of heating at $0.1 \text{ W m}^{-2}$ would suffice to warm only the uppermost $\approx 100 \text{ m}$ of the body by $4 \text{ K}$. The output power to a geyser of given size may nonetheless be comparable to that given above. This may be seen by considering a finite layer of highly permeable nitrogen underlain by impermeable nitrogen or water ice, either of which is effectively a perfect insulator by comparison. Assume the thin layer, with thickness $Z \approx 100 \text{ m}$, has achieved a steady thermal state. The area over which the geyser receives energy from the collector is now reduced to something proportional to $R_{g} Z$, but the potential temperature drop $\Delta \Theta_{\text{max}}$ now occurs across a distance of the order of $Z$, so that the power is roughly the same. In addition, the collector size required to achieve a temperature of $41 \text{ K}$ is reduced as $Z$ is made small. Similar results may well obtain if the geyser taps a developing thermal boundary layer within a thicker permeable region. (Note that, although a thick nitrogen layer on Triton is likely ruled out on rheologic grounds [8], a highly permeable water-ice layer of arbitrary depth could serve as the geyser reservoir, provided the pore spaces were permeated by nitrogen.)

The models presented here are, at best, only marginally able to explain the observed features of Triton's geysers. The calculated geyser radius is greater than that estimated from the plumes themselves, the 30-km collector radius needed to attain $41 \text{ K}$ is considerably larger than the dark features observed at the bases of the active plumes [1], and the posted permeability of the reservoir layer is extremely high. The solutions to these problems may well lie in consideration of explicitly time dependent geyser models. Such a model, in which a geyser taps a gas reservoir some time after it has started to charge, affords the most obvious way of obtaining a finite plume lifetime (less than the length of the Tritonian summer, after which even a quasi-static geyser would shut off). In this context, it is suggestive that the thermal-diffusion distance calculated using the effective conductivity proposed above and a period of 5 years ranges from $\sim 40 \text{ m}$ (at $37 \text{ K}$) to $\sim 300 \text{ m}$ (at $41 \text{ K}$)—precisely the range of geyser radii obtained in the two estimates above. Furthermore, if the estimated power of $5 \times 10^{4} \text{ W}$ corresponds to a peak rather than to a steady-state output, the requirements for permeability and/or geyser radius may be relaxed somewhat. I am currently investigating time-dependent geyser models to attempt to quantify these assertions. Meanwhile, the absence of recognized active plumes on Triton north of the subsolar latitude remains a powerful stimulus for the investigation of insulation-powered geyser mechanisms.

References Cited


Figure Captions

Figure 1. Effective thermal conductivity as a function of temperature for permeable N\textsubscript{2} with grain sizes of 1 mm, 1 cm, 10 cm, and 1 m (solid curves), impervious N\textsubscript{2} (lower dashed curve, scaled from [2] with T\textsuperscript{-1} dependence), and impervious H\textsubscript{2}O (upper dashed curve, interpolated from [4]).

Figure 2. Potential temperature field for a steady-state model of a dark spot (geyser "collector") on Triton with elevated temperature (black) on a surface disk and ambient temperature (white) on the other boundaries. Distinct gray shades represent 16 equal subintervals of total temperature range. Left edge is axis of cylindrical symmetry.