THEORETICAL ASPECTS OF PHOTOCLINOMETRIC TERRAIN PROFILING ON PHOBOS AND OTHER BODIES. Nicholas D. Efford and Lionel Wilson, Environmental Science Division, IEBS, Lancaster University, Lancaster LA1 4YQ, U.K.

The two classic techniques commonly employed to extract topographic information from spacecraft imaging data—stereophotogrammetry and shadow length measurement—suffer from drawbacks that limit their usefulness in many situations: the former is accurate but computationally expensive, and the construction of stereo pairs requires overlapping coverage of the target body's surface; the latter requires only a single image, but its use is generally restricted to regions comparatively near to the terminator or areas of particularly rugged terrain. Furthermore, calculations yield only the difference in elevation between points at the root and foot of a shadow, which will not necessarily correspond to a meaningful topographic measure such as crater depth.

Photoclinometry, or 'shape from shading', is a more general technique, applicable to single images in which shadowing is minimal or absent, and capable of producing *continuous* topographic profiles. Topography is determined by modelling the influence of tilt relative to some reference surface on the measured reflectance of pixels in a spacecraft image (or, in practice, the *ratio* of measured reflectance to that of a locally flat surface, identically viewed and illuminated). Two forms of photometric model have commonly been used for this purpose—Minnaert's law and the Hapke-Irvine law:

$$r(i,e) = A\mu_0^k\mu^{k-1} \tag{1}$$

$$r(i,e,g) = A\frac{\mu_0}{\mu_0+\mu}f(g) \tag{2}$$

Here, μ_0 and μ denote the cosines of incidence and emergence angles i and e. In Minnaert's empirical formulation, A and k are functions of phase angle, g, and hence they will assume constant values in most spacecraft images. Equation 1 has featured in the application of photoclinometry to Viking images of Mars [1,2,3]. For a semi-empirical Hapke-Irvine law, $f(g)$ is normalised, such that $f(0°) = 1$ and the constant A is equal to twice the normal albedo of the surface at the point under study. Whether $f(g)$ is empirical or theoretical in form, its value is usually constant in a single image; equation 2 thus reduces to a simple Lommel-Seeliger scattering law. This has also been widely used for the purpose of photoclinometry, either on its own [4,5] or when linearly combined with a Lambert scattering law (i.e. equation 1 with $k = 1$) [6,7].

Equations 1 and 2 are not entirely suitable for modelling the photometric properties of real planetary regoliths. A more appropriate model, developed by Hapke [8,9,10], can be written

$$r(i,e,g) = \frac{w}{4\pi}\frac{\mu_0'}{\mu_0'+\mu'}\{[1+B(g)]P(g)+H(\mu_0')H(\mu')-1\}S(i,e,g) \tag{3}$$

Here, w is the average single-scattering albedo of regolith particles. $B(g)$ models the opposition effect and $P(g)$ describes the angular scattering characteristics of an average particle; both are functions of phase angle alone, and thus become constants in a photoclinometric problem (although their actual values will be calculable only if the relevant model parameter values are known). The 'H-functions' account for multiple scattering, and hence are dependent on w. μ_0' and μ' differ from μ_0 and μ, having been corrected for the effects of *unresolved* macroscopic roughness, and $S(i,e,g)$ is a function modelling the shadowing caused by this roughness [9]. μ_0', μ' and S all depend on $\bar{\theta}$, a parameter corresponding to the mean slope angle of unresolved topography.

Adopting Hapke's equation as a photometric model, we have generated synthetic quantised reflectance scans across hypothetical topographic features in order to investigate the effects of various errors and invalid assumptions on the topography recovered by photoclinometry. Regoliths with $w = 0.1$ or 0.95 and a range of roughnesses ($0° \leq \bar{\theta} \leq 40°$) are considered here. Figure 1 shows crater profiles recovered photoclinometrically, assuming equation 2, for macroscopically-smooth ($\bar{\theta} = 0°$) low and high-albedo surface material. Predictably, good results are obtained for $w = 0.1$, but the influence of multiple scattering at high albedos leads to a 30% overestimate of crater depth. Figure 2 shows a similar plot, this time for dark ($w = 0.1$), macroscopically-rough surface material. Significant errors are once more apparent, arising because equation 2 does not model roughness effects; their magnitude increases with increasing $\bar{\theta}$. Naive attempts to correct for the skew in the profiles of figures 1 and 2 by varying the assumed flat surface reflectance will level the topography, but will remove neither the distortion to crater shape nor the errors in crater depth.

Experiments have also been performed assuming Minnaert's law. It was found that values of k slightly greater than 0.5 gave acceptable results for dark, smooth material, whilst values nearer unity (corresponding to quasi-lambertian scattering) proved to be more appropriate for a smooth, high-albedo surface. Similar results can be expected for a linear combination of Lommel-Seeliger and Lambert functions. Both formulations cope with high albedos by allowing empirical corrections to be made for multiple scattering. With the introduction of roughness, however, they break down: profiles recovered using equation 1 were found to be severely distorted, with crater depth errors approaching +50%.

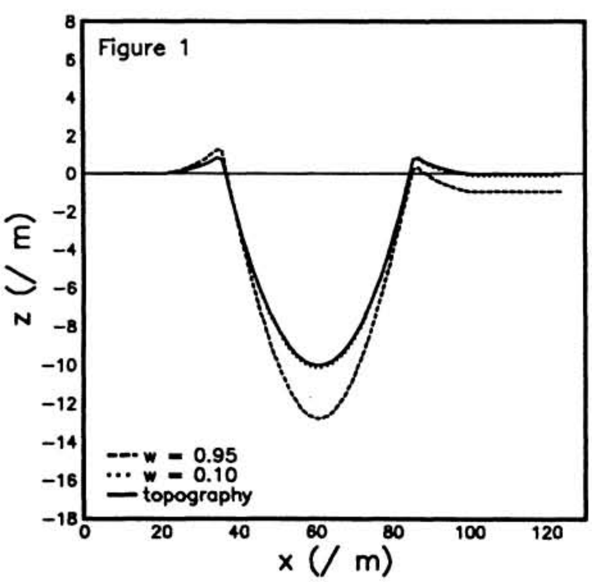
We would advise against the adoption of equations 1 or 2 for the purpose of photoclinometry, and suggest equation 3 as a more suitable alternative. In the case of bodies with very low albedos—such as Phobos, for which w has been determined, from integral photometry of Viking images [11], to be around 0.09—equation 3 may be simplified by collecting together the purely phase angle-dependent terms and neglecting the term in H^2. Hence

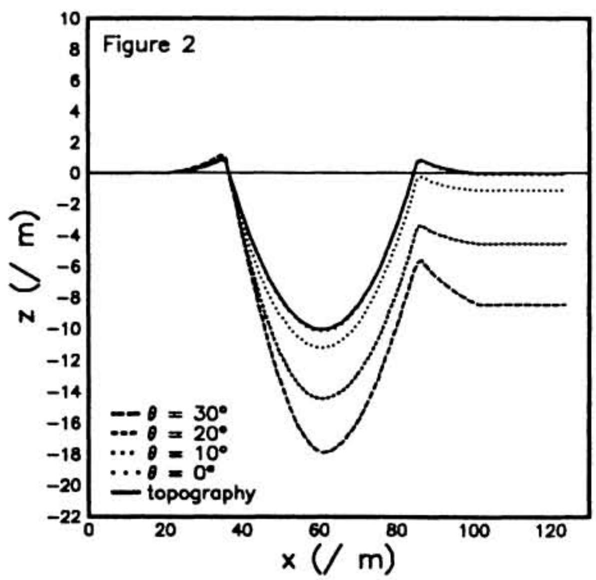
$$r(i,e,g) = A\frac{\mu_0'}{\mu_0' + \mu'}S(i,e,g) \tag{4}$$

where A is a constant which can be thought of as the effective albedo of the pixel under study. The reflectance ratio equation often required in practical photoclinometry can then be written

$$R = \frac{\tilde{r}}{r} = \frac{\tilde{\mu}_0'(\mu_0' + \mu')\tilde{S}}{\mu_0'(\tilde{\mu}_0' + \tilde{\mu}')S} \tag{5}$$

the tilde denoting quantities calculated for the tilted surface. This equation cannot be reduced to a simple analytic form giving tilt as a function of R; tilt is therefore best determined by an iterative procedure, or by the initial generation of a tilt-reflectance ratio look-up table which may be rapidly accessed during profiling to obtain tilt estimates for each pixel along a reflectance scan.





REFERENCES: [1] Davis, P.A. and L.A. Soderblom (1984) *J. geophys. Res.* **89**, 9499–9457. [2] Howard, A.D., K.R. Blasius and J.A. Cutts (1982) *Icarus* **50**, 245–258. [3] Pike, R.J. and P.A. Davis (1984) *Lunar planet. Sci. XV abstract*, 645–646. [4] Jantunen, H. and J. Raitala (1983) *The Moon and Planets* **29**, 7–13. [5] Davies, A.G. and L. Wilson (1987) *Lunar planet. Sci. XVIII abstract*, 221-222. [6] Squyres, S.W. (1981) *Icarus* **46**, 156–168. [7] McEwen, A.S. (1985) *Lunar planet. Sci. XVI abstract*, 528–529. [8] Hapke, B. (1981) *J. geophys. Res.* **86**, 3039–3054. [9] Hapke, B. (1984) *Icarus* **59**, 41–59. [10] Hapke, B. (1986) *Icarus* **66**, 270–279. [11] Efford, N.D. (1989) *Lunar planet. Sci. XX abstract*, 262–263.