THE SHADOW FUNCTION OF STATISTICALLY ROUGH SURFACE: 
THE COMPUTER SIMULATION.

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Shadowing is one of the most significant factors forming observable 
optical properties of statistically rough surface. But neither astronomical 
observations nor laboratory measurements don't allow to separate shadow 
effects from other ones. That's why it is necessary to investigate shadowing 
by theoretical and model methods.

We shall describe shadowing of the surface by one-ray shadow function 
\( F(\theta, \sigma, R) \) and two-ray function \( F(\theta_1, \theta_2, \delta, \sigma, R) \). The \( F(\theta, \sigma, R) \) function is the 
probability that the given point is illuminated (or observed) if light source 
(observer) is situated at \( \theta \) angle over the horizon. Similarly, two-ray 
function \( F(\theta_1, \theta_2, \delta, \sigma, R) \) describes the probability that the given point 
simultaneously is illuminated and observable if angles of illumination and 
observation are \( \theta_1 \) and \( \theta_2 \) and azimuths of rays differ in angle \( \delta \) (Fig. 1).

As it is easy to see, standard deviation \( \sigma \) of the surface height and 
correlation radius \( R \) may effect on \( f \) and \( F \) only in relation \( \mu = \sigma / R \). This 
relation has simple physical sense - standard deviation of surface slopes.
Defining \( A = \tan \theta / \mu \) we can decrease number of variables and simplify 
\( f \) and \( F \) to \( f(A) \) and \( F(A_1, A_2, \delta) \).

\( F(A_1, A_2, \delta) \) function may be calculated analytically only approximately 
in some special cases [1]:
- low shadowing (\( A_1, A_2 \gg 1 \))
- high shadowing (\( A_1, A_2 \ll 1 \) and fully correlated relief along rays 
\( \delta = 0^\circ \)) or uncorrelated relief (this case approximately occurs at \( \delta \approx 180^\circ \)).

In general, for \( A_{1,2} \approx 1 \) and \( 0^\circ < \delta < 180^\circ \) we must calculate \( F(A_1, A_2, \delta) \) by 
numerical modeling, similar to carried in paper [2] for one-ray function.

Here we investigate \( F(A_1, A_2, \delta) \) for surface having Gaussian statistics, 
but the method is usable to arbitrary surface statistics. A computer 
simulation consists of such steps:
1. The generation of a random surface \( H(0) \) with a given statistical 
properties (we used round array of 64655 points, Gaussian statistics of 
height and correlation radius \( R = 7 \)).
2. The rotation of the initial surface \( H(0) \) by different azimuth angles \( \beta_i \) 
obtaining rotated relief \( H(\beta_i) \).
3. The calculation of "sunrise angle" \( V(\beta_i) \) (minimal altitude of light 
source when the given point is illuminated) for all points of \( H(\beta_i) \).
4. The rotation of \( V(\beta_i) \) arrays by angles \( 360^\circ - \beta_i \) for standartization of 
their coordinate systems.
5. The building for any pair \( V(\beta_i), V(\beta_k) \) cumulative histogram, equal 
\( F(A_1, A_2, |\beta_i - \beta_k|) \).

The results of simulations are described at Fig.2-4.

References:
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Stankevich D.G. et al

Fig. 1. Geometry of shadowing.

Fig. 2. Sections of $F(A_1, A_2, \delta)$ on levels 0.2, 0.4, 0.6, 0.8 at $\delta = 0^\circ$ and $180^\circ$.

Fig. 3. Plot of $F(A_1, A_2, \delta)$ at $A_1 = A_2$, $\delta = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 180^\circ$.

Fig. 4. Azimuthal dependence of $F(A_1, A_2, \delta)/F(A_1, A_2, 0)$ at $A_1 = A_2 = 0.5\mu$, $\mu$, $0.1\mu$. 

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