# THE SHADOW FUNCTION OF STATISTICALY ROUGH SURFACE: THE COMPUTER SIMULATION.

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Shadowing is one of the most significant factors forming observable optical properties of statistically rough surface. But neither astronomical observations nor laboratory measurements don't allow to separate shadow effects from other ones. That's why it is necessary to investigate shadowing by theoretical and model methods.

We shall describe shadowing of the surface by one-ray shadow function $F(\theta,\sigma,R)$ and two-ray function $F(\theta_1,\theta_2,\delta,\sigma,R)$. The $F(\theta,\sigma,R)$ function is the probability that the given point is illuminated (or observed) if light source (observer) is situated at $\theta$ angle over the horizont. Similarly, two-ray function $F(\theta_1,\theta_2,\delta,\sigma,R)$ describes the probability that the given point simultaneously is illuminated and observable if angles of illumination and observation are $\theta_1$ and $\theta_2$ and azimuths of rays differ in angle $\delta$ (Fig. 1).

As it is easy to see, standard deviation $\sigma$ of the surface height and correlation radius $R$ may effect on $f$ and $F$ only in relation $\mu = \sigma/R$. This relation has simple physical sense - standard deviation of surface slopes. Defining $A = tang\theta/\mu$ we can decrease number of variables and simplify $f$ and $F$ to $f(A)$ and $F(A1,A_2,\delta)$.

$F(A_1,A_2,\delta)$ function may be calculated analyticaly only approximately in some special cases [1]:
- low shadowing ($A_1$, $A_2 \gg 1$)
- high shadowing ($A_1$, $A_2 \ll 1$) and fully correlated relief along rays ($\delta = 0^o$) or uncorrelated relief (this case approximately occurs at $\delta \approx 180^o$).

In general, for $A_{1,2} \approx 1$ and $0^o<\delta<180^o$ we must calculate $F(A_1,A_2,\delta)$ by numerical modeling, similar to carried in paper [2] for one-ray function.

Here we investigate $F(A_1,A_2,\delta)$ for surface having Gaussian statistics, but the method is usable to arbitrary surface statistics. A computer simulation consists of such steps:
1. The generation of a random surface $H(0)$ with a given statistical properties (we used round array of 64655 points, Gaussian statistics of height and correlation radius $R = 7$).
2. The rotation of the initial surface $H(0)$ by different azimuth angles $\beta_i$ obtaining rotated relief $H(\beta_i)$.
3. The calculation of "sunrise angle" $V(\beta_i)$ (minimal altitude of light source when the given point is illuminated) for all points of $H(\beta_i)$.
4. The rotation of $V(\beta_i)$ arrays by angles $360^o-\beta_i$ for standartization of their coordinate systems.
5. The building for any pair $V(\beta_i)$, $V(\beta_k)$ cumulative histogram, equal $F(A_1,A_2,|\beta_i-\beta_k|)$.

The results of simulations are described at Fig.2-4.

References:

[1] Bass F.G., Fuks I.M. Scattering of waves by statistically rough surface. *Moscow: Nauka, 1972, 424 p.* (in Russian)
[2] Brockelman R.A., Hagfors T. *IEEE Trans. AP-14, no.5, 1966, p.621-629*

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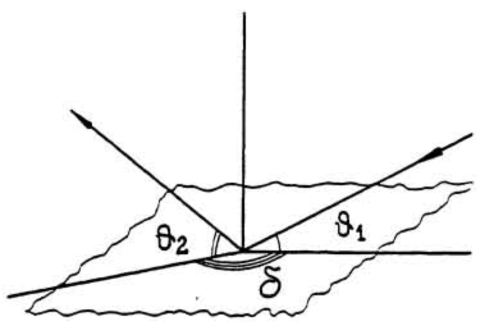


Fig.1. Geometry of shadowing.

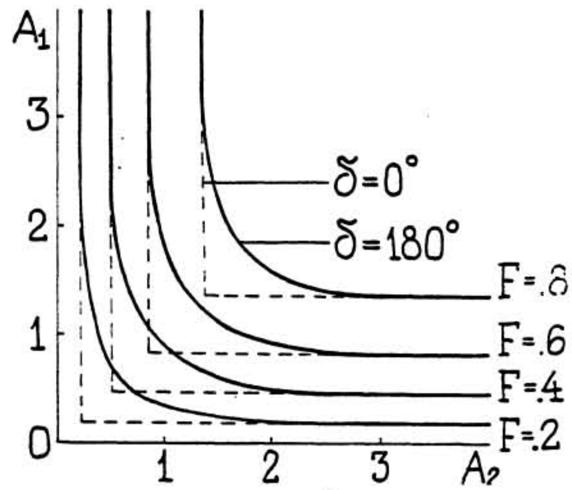


Fig.2. Sections of $F(A_1,A_2,\delta)$ on levels 0.2, 0.4, 0.6, 0.8 at $\delta=0°$ and $180°$.

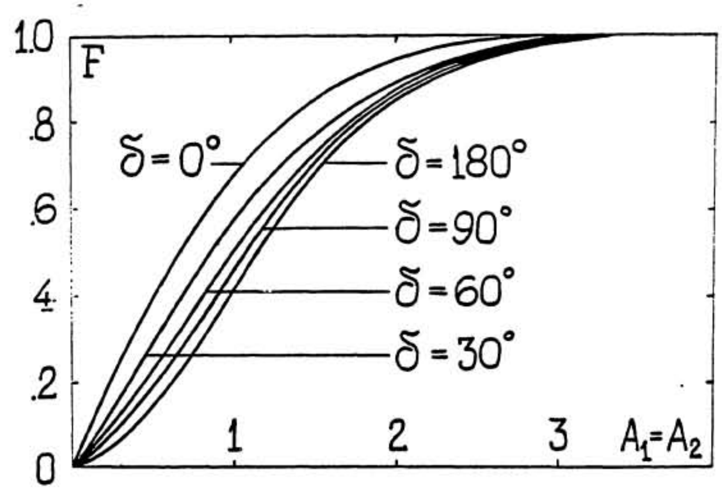


Fig.3. Plot of $F(A_1,A_2,\delta)$ at $A_1 = A_2$, $\delta = 0°, 30°, 60°, 90°, 180°$.

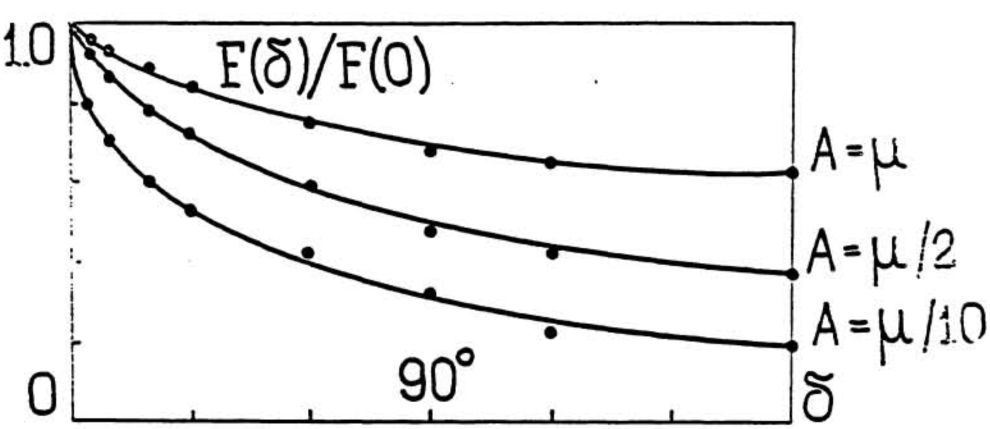


Fig.4. Azimuthal dependence of $F(A_1,A_2,\delta)/F(A_1,A_2,0)$ at $A_1=A_2=0.5\mu$, $\mu$, $0.1\mu$.