

THE THERMAL EROSION OF SINUOUS RILLES: CONSEQUENCES OF DOWN-RILLE VARIATIONS IN LAVA RHEOLOGY, DENSITY, AND FLUID-DYNAMIC FLOW REGIME;

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Background: Hulme's [1] original model of the formation of lunar sinuous rilles as being due to the thermal erosion of a pre-existing surface by a high-effusion rate, turbulent lava flow has been applied by various authors to later measurements of morphological properties of rilles on the Moon [2, 3], Mars [4] and Earth [5]. All of these treatments followed Hulme's approach, using the average rille width to define an allegedly constant volume flux per unit rille width (a value for which can be found from the observed rille length), and using the typical temperature of the lava in the rille (more specifically, the temperature half way along the rille) to find the corresponding Newtonian viscosity and Reynolds and Prandtl numbers, finally using these averaged values to calculate the erosion rate of the rille floor near the vent and hence, from the measured depth, the duration of the eruption and the total volume of lava erupted. We have re-analysed the problem, preferring to work in terms of a constant mass flux of lava erupted from the vent, which allows us to take account of down-rille lava density variations. We then use the measured variations of rille width and depth, deduced variations of lava temperature, viscosity and Reynolds number (obtained as in Hulme's model), and plausible assumed density variations, all as a function of distance from the vent, to calculate in more detail than has been done before the erosion rate and the implied eruption duration for many stations along the rille. All stations should give the same duration, allowing an internal consistency check of the model.

Theory: We assume that a constant mass flux M of lava flows along the floor of a developing sinuous rille channel, the width, w , of which decreases linearly with distance, x , from the source vent, from w_s at the source end to w_d at the distal end, so that

$$w(x) = w_s - \lambda x \quad \dots (1) \quad \text{and} \quad \lambda = [w_s - w_d]/x_m \quad \dots (2),$$

where x_m is the length of the rille channel. We follow Hulme [1] in assuming that the lava loses heat dominantly by radiation from its upper surface, and that the lava motion is turbulent (but see later); by adopting the erupted mass flux as a constant we obtain the heat loss equation

$$dT/dx = -[\sigma \epsilon w(x) T^4]/[cM] \quad \dots (3)$$

where T is the mean lava temperature, σ is Stefan's constant, ϵ is the lava emissivity (taken as unity), and c is the lava specific heat. Integrating, we find the variation of T with distance, x , as:

$$[T^{-3} - T_e^{-3}] = [(3\sigma\epsilon)/(cM)] [w_s x - (1/2) \lambda x^2] \quad \dots (4),$$

where T_e is the lava eruption temperature. Assuming the end of the rille marks the point where T decreases to T_s , the lava solidus temperature (which we take as 1335K[6]), we have:

$$x_m = [(cM)/(3\sigma\epsilon)] [T_s^{-3} - T_e^{-3}] [2/(w_s + w_d)] \quad \dots (5).$$

Thus, if a value is assumed for T_e , we can deduce M from measured values of w_s , w_d and x_m . Hulme [1] gave a method of estimating T_e from the down-channel variation of rille depth, D , and we used this to obtain a first approximation to T_e for each of the 10 rilles we analysed. Once M is known we can find the Reynolds number, R_d , based on the depth, d , of the flowing lava on the rille floor: since

$$R_d = (4 \rho u d)/\eta \quad \dots (6) \quad \text{and} \quad M = \rho u d w \quad \dots (7)$$

where u is the lava velocity, ρ the lava density and η the lava viscosity, it follows that

$$R_d = [(4 M)/(\eta w)] \quad \dots (8),$$

The viscosity of the lava, needed in the above equation, is found from

$$\log_{10} \eta = 15 \log_{10} (1635/T) + 10(21.53512 - 0.01532509 T) \quad \dots (9),$$

where we have combined Hulme's [1] suggested temperature dependence above the liquidus with an empirical fit to data at subliquidus temperatures [7], modified to take account of the systematically low viscosity of lunar basalts. The following equations give the friction factor, f , of the flow:

$$f = [24/R_d] \quad \dots (9) \quad \text{or} \quad (4\sqrt{f})^{-1} = \log(d/e) + 0.57 - \log[1 + 4.67\{(d/e)/(R_d\sqrt{f})\}] \quad \dots (10),$$

(9) being used if $R_d < \sim 1800$ (laminar flow) and (10) if $R_d > \sim 1800$ (turbulent flow). The second equation must be solved recursively, starting from a trial value of $f = 0.01$, and assumes that the rille floor roughness scale, e , is small (~ 5 cm). Once f is known it is trivial to find u and d from

$$u = [(2 g \alpha d)/f]^{1/2} \quad \dots (11) \quad \text{and} \quad d = [f/(2g\alpha)]^{1/3} [M/(\rho w)]^{2/3} \quad \dots (12),$$

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where α is the measured slope of the rille floor. Next, we calculate the heat transfer rate from the rille lava into the rille floor which leads to the floor erosion rate, dD/dt :

$$dD/dt = [0.023 k N Pr^{0.4} R_d^{0.8/4(1+N)} \rho L d] [T-T_s] \quad \dots (13),$$

$$dD/dt = [k N/(1+N) \rho L (4\kappa t)^{1/2}] [T-T_s] \quad \dots (14),$$

where (13) is used for turbulent flow & (14) for laminar flow; k is the lava thermal conductivity, $Pr=[\eta c/k]$ is the Prandtl number, κ is the lava thermal diffusivity $=[k/\rho c]$, and $N=[L/c(T_s-T_a)]$ where T_a is the ambient temperature, $\sim 300K$. Also, t is the time for which lava has been flowing in a laminar fashion after ceasing to be turbulent. Finally, we find the implied duration of the eruption at each station, τ , by dividing the local depth D by the above erosion rate, dD/dt . If the model fits the measured rille morphology perfectly, the same value will be found for τ from all stations down rille; also, the depth of the flowing lava is expected to be much less than the depth of the rille channel.

Analysis: The table below shows the downstream variation of measured (x , w , D) and deduced (T , η , ρ , R_d , f , d , u , dD/dt , τ) parameters for two of the rilles analysed. The first shows a case where the initial estimate of eruption temperature is relatively good, so that similar eruption duration values are found for all stations downstream (median value ~ 12 days). The second is a case in which the initial eruption temperature estimate is much too small; the duration varies systematically (by a factor >3) with x , and the flowing lava depth is greater than the depth of the rille which should have contained it! In this case (and other similar cases not shown), improvements are made by repeating the analysis with a higher estimate of T_a , which has the general effect of reducing lava viscosities and depths, increasing flow speeds and erosion rates, and making durations more uniform.

References. [1] Hulme, G. (1973) *Mod. Geol.* 4, 107-117. [2] Head, J.W. & Wilson, L. (1981) *Lunar & Plan. Sci.* XII, 427-429. [3] Wilson, L., Illing, D. & Head, J.W. (1985) *Lunar & Plan. Sci.* XVI, 916-917. [4] Wilson, L. & Mouginiis-Mark, P.J. (1984) *Lunar & Plan. Sci.* XV, 926-927. [5] Coombs, C.R., Hawke, B.R. & Wilson, L. (1990) *Proc. Lunar Plan. Sci. Conf.* 20th., 195-206. [6] Taylor, S.R. (1975) *Lunar science: a post Apollo view*, Pergamon, New York. [7] Shaw, H.R., Wright, T.L., Peck, D.L. & Okamura, R. (1968) *Am. J. Sci.*, 266, 225-264.

Table. Measured morphological parameters (x , w and D) and deduced fluid dynamic parameters for two lunar sinuous rilles in the Oceanus Procellarum area. The variables are defined in the text. The units of η , ρ and dD/dT are (Pa s), ($kg\ m^{-3}$) and ($\mu m/s$), respectively.

Rille A: initial eruption temperature taken as 1690 K.

x/km	w/m	D/m	T/K	η	ρ	$\log(R_d)$	$\log(f)$	d/m	$u/(m/s)$	dD/dT	$\tau/days$
1.7	60	60	1679	0.67	750	6.146	-2.456	22.5	14.5	120	5.8
4.2	150	80	1663	0.78	907	5.724	-2.387	11.8	9.7	89	10.5
11.0	190	70	1623	1.12	1065	5.462	-2.347	9.3	8.2	60	13.5
16.6	180	50	1592	1.49	1267	5.342	-2.328	8.7	7.8	43	13.4
22.5	250	30	1562	1.98	1470	5.079	-2.284	6.6	6.4	30	11.6
28.3	220	10	1535	2.57	1672	5.041	-2.276	6.6	6.4	24	4.8
34.1	270	80	1510	3.31	1875	4.851	-2.237	5.6	5.6	17	54.0
38.0	300	10	1493	3.89	2010	4.724	-2.215	5.0	5.2	14	8.5
44.7	280	10	1467	5.09	2235	4.633	-2.194	4.9	5.0	10	11.7
52.9	190	10	1437	6.9	2527	4.690	-2.208	6.0	5.6	7	16.8
61.4	640	5	1409	18.4	2365	3.725	-1.979	2.9	3.0	2	24.9
66.5	260	10	1393	31.1	3000	3.881	-2.018	4.9	4.1	2	69.4

Rille B: initial eruption temperature taken as 1380 K.

x/km	w/m	D/m	T/K	η	ρ	$\log(R_d)$	$\log(f)$	d/m	$u/(m/s)$	dD/dT	$\tau/days$
0.4	40	40	1380	108	735	4.898	-2.252	114	25.6	2.42	191
1.8	140	20	1379	133	1000	4.230	-2.114	43	13.5	1.47	158
4.8	110	20	1377	231	1485	4.114	-2.086	40	12.6	1.01	228
7.8	90	20	1375	276	1992	4.146	-2.092	39	12.4	0.86	269
10.2	110	10	1374	1780	2367	3.232	-1.585	44	7.4	0.24	475
12.0	50	20	1373	1740	2676	3.594	-1.951	51	12.2	0.34	673
13.6	50	10	1372	3810	2941	3.272	-1.638	61	9.3	0.19	599