MODELING MARTIAN CHANNELS BY BIFURCATION THEORY AND DIFFERENTIAL TOPOLOGY: ARGUMENTS FOR DRAINAGE OF SUBSURFACE AQUIFER.
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Topological analysis of martian channel networks through the Horton's laws shows the remarkable insensitivity of their organization to detailed geomorphologic dynamics [1]. The population of martian networks is governed by erosional regressive process producing binary arborescent systems. Statistical investigations by application of Strahler's method have confirmed the validity of this empirical law to martian networks [2]. Binary trees are subject to mathematical properties of distribution by combinatory analysis. The number $B(n)$ of topologically distinct channel networks having $n$ sources may be computed by the recursion formula:

$$B(n) = \sum_{i=1}^{n-1} (B_i).B(n-i)$$

Similarly, the number $B(n,\Omega)$ of topologically distinct channel networks of order $\Omega$ formed by $n$ channels of order $\omega$ is:

$$B(n,\Omega) = \sum_{i=1}^{\Omega-1} [B(i;\Omega-1).B(n-1;\Omega-1)+2B(i;\Omega)\times \sum_{w=1}^{\Omega-1} B(n-1,w)]$$

After slight manipulation, a solution by binary analysis (Catalan number) shows that there are $1.2.3...(2^{n-2})/1.2.3...(n-1)^2n$ binary channel networks of order $n$. For example, there are 14 binary networks of order $\Omega=5$, 42 of order $\Omega=6$... That means that combinatory analysis is capable of simulating the distribution of network branches. Statistical comparison by Horton's graphical method of this property supports the conclusion that all binary networks have identical chance to be the model of natural martian networks [3]. The relationship between the first order branches $n_1$ and the order $n_2$ branches in total $N$ branching system is statistically defined by the equation:

$$N = (.66+/-0.3)n_1 + (.33+/-0.3)n_2 + .01n_3$$

for a network of order $\Omega = 3$. The random property of the first order branches is topologically transmitted to the second order.

The general behaviour of this binary system comprising a high ratio of order 1 branches and which is organized with a minimum of divergences is typical to a limited dynamic system. In the network organization, the distribution of channels can be described by a set of state variables $(X)$ which are the dependant variables to be predict in a model and $(u)$ a set of parameters (or independant variables). The equilibrium position of this system is $f(X,u) = \text{Min}(X,u)$. The dynamics of the process is:

$$-\delta f/\delta X = -\text{grad } f \quad (\text{the minimum occurs when } \text{grad } f = 0)$$

As the system is limited, the potential function $f$ is limited.

In the case of regressive channel network, the potential is the feeding reservoir which is supposed to be a subsurface aquifer localized into the regolite under frozen layers. The space phasis system could be defined by the parameters of the aquifer (porosity, density, temperature...). The process of drainage leads to a variation of the aquifer parameters towards an
MODELING MARTIAN CHANNELS. Grin, E.A, and Cabrol, N.A.

equilibrium position [3]. The space of phasis is contracted and the system of the drainage energy is dissipative. Therefore, this space of phasis is considered as a fractal attractor of the dissipators (channels) of the energy.

The initiation of a branch starts at a signal during the drainage course. At every signal (geological events such as fractures, collapses, marsquakes...) generating a new dissipator, the geometry of the attractor is modified. By recurrence, when the distribution and the nature of the signals are known, the geometry of the attractor could be redefined. The limit of equilibrium of the potential aquifer could be determined by the distribution of the network junction positions. The equilibrium of a limited system for a given state of phasis is defined as a crest of potential for the drainage system determined by all its signals. As soon as the crest of potential is not reached, the network developed itself by upward regression.


Fig.1: The graph shows the theoretic evolution of the dynamic system parameters. $b_1$ and $b_2$ correspond to signals (events) which modify the behaviour of the system.

Fig.2: Application to the genesis of martian networks by drainage of a subsurface aquifer. At every signal (i.e collapse), a new flow emergence is generated.

d is the drained zone ; e is the emergence level ; f is a regressive erosional front. Every transitional potential level of the residual aquifer corresponds to a temporary attractor.