MIGRATION OF BODIES DURING THE ACCUMULATION OF TERRESTRIAL PLANETS. S.I. Ipatov
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Our investigations of the process of the formation of the terrestrial planets as the investigations made by Wetherill [1] were based on the results of computer simulation of the evolution of disks initially consisting of hundreds of identical gravitating solid bodies moving around the Sun and coagulating under collisions. The mutual gravitational influence of the bodies was taken into account by the method of action spheres, i.e. we considered two 2-body problems. Algorithms for calculating the probability \( p_{ij} \) that objects (bodies or planets) \( i \) and \( j \) could encounter at the distance equal to the action sphere radius, and for calculating the moment \( \tau_{ij} \propto 1/p_{ij} \) of the first isolated encounter of these objects as well as the algorithm for simulating the objects encounters were considered in [2]. Initially we used the "probability" algorithm for choosing the pairs of encountering objects. For this algorithm, we stored in computer memory \( N(N-1)/2 \) elements of matrix \( \{ p_{ij} \} \) for which \( j > i \), where \( N \) is the total number of objects. Later on [2], the efficient "deterministic" method was developed for which for a pair of encountering bodies the time before an isolated encounter is minimum. In this method called the method of conditional triangular matrix (CTM) besides the information about objects, we store \( 3N-2 \) numbers: the values of \( \tau_i = \min \{ \tau_{ij} \} = \tau_{ik_i} \), \( k_i \), \( t_i \), and \( t_N \), where \( i = 1, \ldots, N-1 \), \( k_i \) is the number \( j \) corresponding to the minimum of \( \tau_{ij} \) for \( j > i \), and \( t_i \) is the time of the last encounter of the object \( i \) with some other object. Matrix \( \{ \tau_{ij} \} \) is not stored and the number of the calculations of new values of \( \tau_{ij} \) which are made after one encounter is small. If we don't use the pseudo-random numbers for calculations of \( \tau_{ij} \), then using the CTM method we obtain the same pairs of encountering objects as using the method of "full search" in which before each encounter the entire matrix \( \{ \tau_{ij} \} \) is calculated and its minimum element among \( N^2 \) elements of the matrix is determined. The time of calculations of the above disks obtained with the use of the CTM method was 1.5–2 times less than that obtained with the use of the method of virtual contacts considered by Eneev and Kozlov [3].

Let us consider the results of two computer runs obtained with the use of the CTM method. Initial data for these runs were identical but different pseudo-random numbers were used for choosing initial orientations of orbits. The initial masses of the bodies were assumed to be equal to 0.002\( m_\oplus \), where \( m_\oplus \) is the mass of the Earth, and their orbits were almost circular. In order to investigate the mixing of bodies during the evolution, all \( N_0 = 960 \) initial bodies were divided into four groups depending on the values \( a_k^0 \) of their semimajor axes. It was considered that
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\[ a_k^0 = \sqrt{(a_{\text{max}}^i)^2 - (a_{\text{min}}^i)^2} \cdot k/N_i + (a_{\text{min}}^i)^2, \]

where \( k = 1, \ldots, N_i \) and \( N_i \) is the number of bodies in the group \( i \). The values of \( a_{\text{max}}^i \) at \( i = 1, 2, 3, \) and 4 were equal respectively to 0.6, 0.8, 1.0, and 1.2 AU, and \( a_{\text{min}}^i = 0.4 \) AU. It was obtained that if the mass of the formed planet is not small then the ratio of the total mass \( m_k \) of bodies of the group \( i \) entered in the planet to the mass \( m_k \) of this planet is close to \( N_i / N_0 \) and almost doesn't depend on the value of the semimajor axis of the planet. For example, we have \( |m_{k_i} / m_k - N_i / N_0| \leq 0.03 \) at \( m_k \geq 0.4m_\oplus \) and \( |m_{k_i} / m_k - N_i / N_0| \leq 0.09 \) at \( m_k \geq 0.17m_\oplus \). Such a strong mixing was obtained by Wetherill [1] only for Venus. The mixing of bodies can be weaker if we take a considerably larger number of bodies and involve the gas resistance. Masses, semimajor axes, and eccentricities of the formed planets were almost the same as those obtained by using the "probability" algorithm for choosing encountering bodies [1,4]. In particular, the orbital eccentricities of the planets formed at the disk edges were larger than those of the planets formed in the center of disk. These results show that the orbital eccentricities of Mercury and Mars could have achieved the large present values due to encounters of these planets with the bodies from the feeding zone of the terrestrial planets. Some bodies entered into the asteroid belt during the evolution. About 9% of all initial bodies for the first variant and 14% for the second variant were ejected into hyperbolic orbits. The accumulation times \( T \) for the main mass of the terrestrial planets have been \( \sim 10^6 - 10^7 \) years and some bodies could have fallen on these planets after more than \( 10^7 \) years had passed. The above values of \( T \) are about ten-fold smaller than the accumulation times obtained by using the "probability" algorithm [1, 4-5].

The computer simulation results and the analytical investigations of the evolution times for disks and the changes in average eccentricities of orbits of bodies depending on the number of disk bodies and their masses allowed some estimates of the evolution of disks initially consisting of a large number of bodies [4-5]. As the last stages of the accumulation took the largest part of the whole time, the estimates of the total evolution time of the disks consisting of a large number of bodies are close to the results obtained for \( N_0 = 960 \). The average orbital eccentricity of the bodies during evolution may have exceeded 0.2 and the number of large \((\geq 0.05m_\oplus)\) embryos of the terrestrial planets could have been greater than the actual number of the planets. It is difficult to obtain the period of axial rotation of the Earth and the inclination of its orbit without taking into account these embryos.