WHY CONVECTIVE HEAT TRANSPORT IN THE SOLAR NEBULA WAS INEFFICIENT; P. Cassen, NASA-Ames Research Center, Moffett Field, CA 94035.

The radial distributions of the effective temperatures of circumstellar disks associated with pre-main sequence (T Tauri) stars are relatively well-constrained by ground-based and spacecraft infrared photometry and radio continuum observations [see the compilation in (1)]. If the mechanisms by which energy is transported vertically in the disks are understood, these data can be used to constrain models of the thermal structure and evolution of solar the nebula (2).

Several studies of the evolution of the solar nebula have included the calculation of the vertical transport of heat by convection (3, 4, 5, 6). Such calculations rely on a mixing length theory of transport and some assumption regarding the vertical distribution of internal dissipation. In all cases, the results of these calculations indicate that transport by radiation dominates that by convection, even when the nebula is convectively unstable. We present here a simple argument that demonstrates the generality (and limits) of this result, regardless of the details of mixing length theory or the precise distribution of internal heating. It is based on the idea that the radiative gradient in an optically thick nebula generally does not greatly exceed the adiabatic gradient.

A well-known result from radiative transfer theory for a plane-parallel, optically thick medium, in which the flux is constant, is that the temperature at depth is related to the effective temperature $T_e$ by

$$ T = T_e (\frac{3}{4} \tau_0)^{1/4}, $$

where $\tau_0$ is the optical depth of the layer, defined by

$$ \tau_0 = \int_0^\infty \rho \kappa dz = \kappa \Sigma / 2, $$

$\kappa$ is the mass averaged Rosseland mean opacity, and $\Sigma / 2$ is the surface density of the layer (half nebula thickness). In general, one may write

$$ T_m = T_e (\eta \tau_0)^{1/4} $$

where $T_m$ is the midplane temperature and $\eta$ is a number less than (but of order) unity for smoothly distributed energy sources. (For instance, $\eta = 3/8$ if the energy sources are uniform with respect to the volume density of the layer.)

The corresponding relationship for an adiabatic temperature profile can be found from the expression for the vertical distribution of temperature:

$$ T = T_m (1 - z^2/h^2), $$

where

$$ h = \frac{\Omega}{c_s} \sqrt{\frac{\gamma - 1}{2}}, $$

$\Omega$ is the local orbital frequency, $\gamma$ is the ratio of specific heats, and $c_s$ is the midplane sound speed. The scale height $h$ is related to the nebula surface density $\Sigma$ and midplane volume density $\rho_0$ by

$$ \Sigma = 2 \rho_0 h \int_0^1 \frac{1}{(1-u^2)^{\gamma-1}} du = \zeta \rho_0 h, $$
where the value of the integral $\zeta = 0.982$. The effective temperature is given by $T_e = T(\tau = 2/3)$. For an optically thick nebula, one expects that the $\tau = 2/3$ level is close to $h$, in which case

$$2 = \int \frac{1}{1-e} h \kappa \rho (1 - u^2) \gamma(\gamma - 1) \, du = \frac{\varepsilon}{\gamma} \int h \kappa \rho (2x)^{1/\gamma} \gamma(\gamma - 1) \, dx = \frac{\gamma - 1}{\gamma} h \kappa \rho (2\varepsilon)^{\gamma(\gamma - 1)}.$$

Here, $\varepsilon = 1 - (z_e/h) \ll 1$, $z_e = z(\tau = 2/3)$, and $\kappa$ is the Rosseland mean opacity near $\tau = 2/3$. This expression yields the result

$$T_m = T_e \frac{[3(\gamma - 1)\kappa \Sigma 4 \gamma_s^2 (\gamma - 1)\gamma]}{[3(\gamma - 1)\kappa \Sigma 2 \gamma_s^2 (\gamma - 1)\gamma]} = T_e \frac{[3(\gamma - 1)\kappa \Sigma 2 \gamma_s^2 (\gamma - 1)\gamma]}{[3(\gamma - 1)\kappa \Sigma 2 \gamma_s^2 (\gamma - 1)\gamma]},$$

Therefore, the ratio

$$\frac{T_m(\text{radiative})}{T_m(\text{adiabatic})} = \frac{\eta^{1/4}}{3(\gamma - 1)\kappa \Sigma 2 \gamma_s^2 (\gamma - 1)\gamma} \approx \frac{\eta^{1/4}}{3(\gamma - 1)\kappa \Sigma 2 \gamma_s^2 (\gamma - 1)\gamma} \approx 0.03 \text{ for } \gamma = 7/5.$$

In the temperature range 160 - 1500 K, the Rosseland mean opacity decreases due to the evaporation of opacity-producing solids, but increases with temperature in those ranges within which no important solid species evaporates (7). The result is that $\kappa \Sigma \kappa$ will commonly be less than one, and in no case (for $50 K < T < 1500 K$) be greater than about three. Thus the midplane temperature that is obtained for conditions of radiative equilibrium cannot be substantially modified by convection, which could only reduce it to the adiabatic value, at best.

The significance of this result is that the problem of understanding the thermal structure of observed circumstellar disks is reduced to a one-parameter ($\tau_0$) issue. In general, one may then define the problem of inferring nebula thermal evolution from observations of T Tauri stars in terms of the determination of only two parameters, the mass accretion rate from the disk to the star $\dot{M}$ and the optical depth $\tau_0$. To see this, note that for disks in which the effective temperature is well-represented by a power law in radius (the usual case), conservation of energy, quite generally, requires

$$\frac{GM_* \dot{M}}{2R_*} = \int_{R_*}^{\infty} 4\pi \rho \sigma T_l^4 (r/r_1) \, dr.$$

The quantities $M_*, R_*, T_l(r_1)$, and $q$ can all, in principle, be determined from observations, and therefore so can $\dot{M}$ [see, for instance, (8)]. One can, for example, derive the result (for $q = 3/4$) that the requirement that silicates be melted at the midplane at 1 AU is:

$$\dot{M} \tau_0 \geq 4 \times 10^{-3},$$

where $\dot{M}$ is measured in solar masses/year. At present, determination of the optical depth $\tau_0$ requires coagulation theory.