PRECESSION OF URANUS AND NEPTUNE AND THEIR MAGNETIC FIELD

Sh.Sh. Dolginov 142092. IZMIRAN Troitsk Moscoov,Reg.RF.

The strength of the dipole magnetic fields of a planet, 

\(H_p\), can be estimated, relatively to that of the Earth at the 
epoch of the observation, \(H_e\), as follows:

\[H_p = H_e \left( \frac{G_e^3 R_e^3 \omega_p \Omega_p \sin \alpha_p \rho_p \sigma_p}{G_p^3 R_e^3 \omega_e \Omega_e \sin \alpha_e \rho_e \sigma_e} \right)^{1/2} \]

\[\Omega = 1.5 M G f \cos \Omega \rho \right)^{1/2} \]

where \(\omega, \Omega, \alpha, \rho, \sigma, f\) are the angular velocities of rotation 
and precession, the angle between \(\bar{w}\) and \(\bar{n}\) the density, 
electric conductivity and dynamic flattening the planets, 
respectively. \(M\) and \(r\) are the mass and the distance to the body 
causing the precession. \(Q = R_n / R_c\) with \(R_n\) the planet radius and 
\(R_c\) the upper boundary of the magnetically active region. For 
Uranus and Neptune, \(R_c\) can be determined from the spectrum of 
Gaussian harmonics.

\[C_n = (2n+1)^{-1/2} (\frac{R_n}{R_c})^{n+2} \sum [g_n^m + h_n^m]^1/2 \]  

from the following condition: at \(r = R_c\), \(C_1 - C_2 = 0\), where \(C_1\) 
and \(C_2\) are the moduli of the dipole and quadrupole harmonics, 
respectively \([1,2]\). The parameters entering eqs. (1) and (2) 
are given for Uranus and Neptune in Refs.\([3,4-7]\) basing on the 
Voyager 2 results. The values of \(\sigma\) have been obtained from 
laboratory studies for the Uranus and Neptune materials \([8]\).

Table 1, The determination of \(R_c\) for Neptune and Uranus

\| \(C_{N1}\) | 12244 nT | \(C_{N2}\) | 14222 nT | \(C_{U1}\) | 22837 nT | \(C_{U2}\) | 45967 nT + \(\Delta\) |
\hline
\(r_{Nkm}\) | 20000 9500 19200 19150 19100 19000 18500 18000 |
\hline
\(\Delta C_{12N}\) | 564 297 57 0 -15 -123 -694 -1317 |

Similar arguments yield \(R_c = 13800\) km for (Uranus) and \(R_c = 
14700\) km for (Uranus 2)

The generation of magnetic fields in Uranus and Neptune 
occurs at very different depths, at very different values of \(\sigma\) 
This assertion is confirmed by the estimations of the Reynolds 
number \(R_m\) and agrees with the difference of the contributions 
of the Joule heat losses into the observed heat fluxes of 
Uranus and Neptune (see below).
The parameters in eq(1) and eq(2)

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>EARTH</th>
<th>NEPTUNE</th>
<th>URANUS 1</th>
<th>URANUS 2</th>
<th>PRECESSION RATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_m$, km</td>
<td>6271</td>
<td>24765</td>
<td>25600</td>
<td>25600</td>
<td></td>
</tr>
<tr>
<td>$R_o$, km</td>
<td>3460</td>
<td>19150</td>
<td>13600</td>
<td>14700</td>
<td></td>
</tr>
<tr>
<td>$\beta$ $m^{-1}$</td>
<td>10</td>
<td>1.9</td>
<td>3.4</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>23.5</td>
<td>21</td>
<td>Tab. 1</td>
<td>Tab. 1</td>
<td></td>
</tr>
<tr>
<td>$\Omega/10^5$ $s^{-1}$</td>
<td>7.27</td>
<td>10.63</td>
<td>10.12</td>
<td>10.12</td>
<td></td>
</tr>
<tr>
<td>$\Omega^*/$ $year$</td>
<td>50.2</td>
<td>40.39</td>
<td>Tab. 2</td>
<td>Tab. 2</td>
<td></td>
</tr>
<tr>
<td>$\sigma$ $m^{-1}$</td>
<td>3 $10^5$</td>
<td>1.2 $10^5$</td>
<td>0.6765</td>
<td>0.6765</td>
<td></td>
</tr>
<tr>
<td>$f_m$ measured</td>
<td>30320</td>
<td>14240</td>
<td>22840</td>
<td>22840</td>
<td></td>
</tr>
<tr>
<td>$f_m$ calculated</td>
<td>14240</td>
<td>16665</td>
<td>22915</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\Omega^* = V_0 \sin \alpha \cdot R_c / 2f$

Table 2: The parameters in eq(1)
Table 3: The parameters in eq(2)

(a) For $R_c = 19150$ km and $\sigma = 1.20 m^{-1}$, we have for Neptune $R_m = 32 > R_m cri$, whereas for Uranus we obtain $R_m = (4.8 - 12) < R_m cri$, where $R_m cri$ is the critical value of $R_m$ at which the magnetic field generation is still possible (see eq.(3) and Table 2).

(b) The Joule heat losses associated with the dynamo activity are inversely proportional to the electric conductivity squared, [11]. For the strength of the Uranus field within the planet being four times the Neptune field strength, the losses in the magnetically active region of Uranus can be $2.5 \times 10^3$ times smaller than in the Neptune since $\sigma_U = 10^2 \sigma_N$ (see Table 2).

References.