FORMATION OF PERCHED LAVA PONDS ON BASALTIC VOLCANOES: INTERACTION BETWEEN COOLING RATE AND FLOW GEOMETRY ALLOWS ESTIMATION OF LAVA EFFUSION RATES. L. Wilson¹,² and E. A. Parfitt¹. ¹Geological Sciences Department, Brown University, Providence, RI 02912. ²Environmental Science Division, Lancaster University, Lancaster LA1 4YQ, U.K.

Perched lava ponds are infrequent but distinctive topographic features formed during some basaltic eruptions. Two such ponds, each ~ 150 m in diameter, formed during the 1968 eruption at Nāpāu Crater [1] and the 1974 eruption of Mauna Ulu [2], both on Kīlauea Volcano, Hawai‘i. Each one formed where a channelised, high volume flux lava flow encountered a sharp reduction of slope; the flow spread out radially and stalled, forming a well-defined terminal levee enclosing a nearly circular lava pond. We describe a model of how cooling limits the motion of lava spreading radially into a pond and compare this with the case of a channelised flow. The difference in geometry has a major effect, such that the size of a pond is a good indicator of the volume flux of the lava forming it. Lateral spreading on distal shallow slopes is a major factor limiting the lengths of lava flows.

Theoretical Considerations: If a Newtonian lava with viscosity μ and bulk density ρ moves with mean speed U and thickness D down a plane inclined at α to the horizontal, the equation of motion in the x coordinate direction is

$$U \frac{\partial U}{\partial x} = g \sin \alpha - \left( \frac{\mu}{D^2} \rho \right)$$  

(1)

where g is the acceleration due to gravity and K is a constant which depends on the cross-sectional geometry of the flow. For flows which are much wider than they are deep, K = 3, whereas for flows in a semicircular channel, K = 8 [3]. Eq. (1) assumes that a steady state prevails, i.e. there are no changes of velocity with time.

The lava motion is found by combining (1) with a continuity equation expressing conservation of volume of the flowing material. For a flow in a channel of uniform width the volume flux $V_C$ is:

$$V_C = U_C D_C W_C$$  

(2)

where $W_C$ is the channel width, and $U_C$ and $D_C$ are the velocity and flow depth in the channel geometry case. The channel width does not change with x, so $U_C$ and $D_C$ are constant, and eq. (1) simplifies to

$$U_C = \frac{D_C^2 \rho g \sin \alpha}{(K \mu)}$$  

(3).

Each of $U_C$ and $D_C$ can be eliminated in turn between eqs. (2) and (3) to give:

$$D_C = \left( \frac{(W_C K \mu)}{(W_C \rho g \sin \alpha)} \right)^{1/3}$$  

(4a),

$$U_C = \left( \frac{(W_C^2 \rho g \sin \alpha)}{(W_C^2 K \mu)} \right)^{1/3}$$  

(4b).

The time, $t_C$, needed for a given batch of lava in the flow to travel a distance $x_C$ is given by

$$t_C = \frac{x_C}{U_C}$$  

(5),

since the velocity $U_C$ is constant. Eliminating $U_C$ with eq. (2) we have:

$$t_C = \frac{(D_C W_C x_C)}{V_C}$$  

(6).

For a flow in a semicircular pond the continuity relation is:

$$V_P = \pi x_P U_P D_P$$  

(7)

where the velocity and depth are $U_P$ & $D_P$, respectively, at a distance $x_P$ from the centre of curvature. Since $V_P$ now involves a dependence on $x_P$, it follows that both $U_P$ and $D_P$ will be functions of $x_P$, and so in general it is no longer possible to neglect the term $[U (\partial U/\partial x)]$ in eq. (1). However, we have integrated eqs. (1) and (7) simultaneously using numerical methods and find that, over a wide range of values of $V_P$ and $x_P$, analytical solutions obtained neglecting the term $[U (\partial U/\partial x)]$ differ by less than 1% from numerical solutions. Expressions for the variations of $U_P$ and $D_P$ with $x_P$ are found by eliminating each of $U_P$ and $D_P$ in turn between eqs. (3) and (7), giving:

$$D_P (x) = \left( \frac{(V_P K \mu)}{(\pi x_P \rho g \sin \alpha)} \right)^{1/3}$$  

(8a),

and

$$U_P (x) = \left( \frac{(V_P^2 \rho g \sin \alpha)}{((\pi^2 K \mu x_P^2)} \right)^{1/3}$$  

(8b).

Both $U_P$ and $D_P$ decrease with increasing $x_P$, with $U_P$ showing the most rapid change. Since $U_P$ is a function of $x_P$, the time $t_P$ needed for a batch of lava to travel outward from its source is found from:

$$dx = U_P dt$$  

(9).

Substituting eq. (8b) for $U_P$ and integrating from an initial small radius $x_P0$:

$$t_P = \frac{3}{5} \left( \frac{(\pi^2 K \mu)}{(V_P^2 \rho g \sin \alpha)} \right)^{1/3} \left( x_P^{5/3} - x_P0^{5/3} \right)$$  

(10).

Since $x_P0$, having a value of at most a few metres, will generally be very much smaller than the distance at which...
the flow stops, we can approximate (10) by

\[ t_p = \left( \frac{3}{5} \right) \left[ (\pi^2 K \mu) / (V_p^2 \rho g \sin \alpha) \right]^{1/3} \times p^{5/3} \]  

(11).

By analogy [4] with engineering data on the cooling of a fluid in a pipe, motion of a lava flow stops when a thermal cooling wave has penetrated from the flow surface to a distance which is some critical fraction \( q^2 \) of the flow thickness \( D \). In time \( t \), a cooling wave penetrates a distance \( \lambda = (\pi t)^{1/2} \) as long as heat transfer is entirely limited by the thermal diffusivity, \( \kappa \), of the flow material, rather than by the ability of the environment to remove heat. This is a good approximation at times more than a few minutes after a given batch of material has been erupted [5], so the time to cessation of flow, \( T \), is

\[ T = (q^2 D^2) / \kappa \]  

(12),

where \( \kappa \) is \( 10^{-6} \) m²/s for all silicates. Field data show that \( q^2 \) has a value close to \( (1/300) \) [6]. This model ignores lava rheology changes as a function of distance from the vent and hence time since eruption. Instead, it establishes a correlation, with the theoretically expected functional form, between maximum flow length and effusion rate. The changes in rheology, and consequent deceleration and eventual cessation of motion, are all included through the value used for \( q^2 \). We can now find cooling-limited travel distances for the above flow geometries. For flow in a channel, we equate \( T \) given by eq. (12) to \( t_c \) given by eq. (6). Identifying \( D \) in equation (12) with \( D_c \) given by eq. (4a) and writing \( x_c = X_c \) for the distance travelled:

\[ X_c = \left[ 1/(300 \times \xi) \right] \left[ V_c / W_c \right]^{4/3} \left[ (K \mu) / (\rho g \sin \alpha) \right]^{1/3} \]  

(13).

For flow in a semicircular pond, we equate \( T \) from eq. (12) to \( t_p \) in eq. (11), explicitly identify \( D \) in eq. (12) with \( D_p \) from eq. (8a) and solve for \( x_p \). Putting \( x_p = X_p \) when the motion stops,

\[ X_p = \left[ (V_p^4 K \mu) / (26 \times 36 \times 53 \times \pi^4 \times \kappa^3 \rho g \sin \alpha) \right]^{1/7} \]  

(14).

Discussion

Eqs. (13) and (14) show the relatively weak dependence of flow length on the lava properties \( \mu \) and \( \rho \), the environmental factors \( g \) and \( \alpha \), and the geometric parameter \( K \), especially in the case of motion in a pond; the most important control on flow length is the volume flux \( V \). We have calibrated eq. (14) above by comparing the predicted and observed eruption rates of the events producing the Mauna Ulu and Napau ponds on Kilauea [7]. The Mauna Ulu pond has a radius of 70-80 m which implies an eruption rate of 12.6-16.0 m³/s. Thus \( X_c \) is 103 km whereas \( X_p \), the maximum distance travelled by an unchannelised (ponded) flow, is only 339 m. Thus if a previously channelised flow encounters topography which causes or allows lateral spreading, flow thinning caused by the change in geometry can produce a dramatic difference in the subsequent possible travel distance. We argue that this is the main cause of the development of perched lava ponds (and of the cessation of advance of many lava flows). The influences of \( \alpha \), \( g \), and \( \mu \) are so weak (seventh root in the case of radial motion, cube root for channelised flows) that \( X_c / X_p \) will always be \( \sim 300 \) for maif lavas with \( V \sim 200 \) m³/s. The dependance of \( X_c / X_p \) on \( V \) in the table below shows that topographic spreading will be particularly important for very high effusion rates such as those postulated [10] to explain the long lava flows found on Venus.

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<th>2</th>
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<th>10</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>200</th>
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References