NON-LINEAR DISPERSION OF SPIRAL DENSITY WAVES IN SATURN'S RINGS; Linda J. Horn and Nicole J. Rappaport (formerly Borderies), Jet Propulsion Laboratory, Pasadena, CA 91016

We have analyzed the dispersion behavior of a number of spiral density waves in Saturn's A ring using the non-linear dispersion relation [1]. We demonstrate that the dispersion behavior of the Mimas 8:5 and Prometheus 7:6 density waves is linear. On the other hand, the Mimas 5:3 density wave in Saturn's A ring disperses non-linearly over the extent of the wave and displays an unexpected difference in surface mass density between the Voyager radio science (RSS) and Voyager photopolarimeter (PPS) occultation data sets. The RSS surface mass densities were larger than the PPS values. This inconsistency may be indicative of different processes affecting the large (RSS) and small (PPS) particle populations. The dispersion behavior of these waves provides an important tool for probing the physical characteristics of the rings and estimating the ring age (which is currently thought to be much younger than the age of the solar system).

The theory of spiral density waves was first developed to explain the spiral structure of some galaxies [2]. For planetary rings it was predicted that density waves would be excited by gravitational interactions with satellites [3]. Spiral density waves are generated at resonance locations, the ring radius where the ring particle and satellite orbital periods are essentially in an integer ratio (i.e. 2:1). Under these conditions the ring particle experiences a resonant response to the periodic forcing from the satellite's gravity. This forcing perturbs the ring particle orbits. Energy and angular momentum are transferred between the rings and the satellites. For satellites external to the rings the energy is transferred from the ring to the satellites and the satellites move away from the rings. Spiral density waves appear in the rings as spiral arms of alternating compressed and rarefied ring material that orbit the planet in the equatorial plane. The radial scales of the density waves range from ~300 m to 300 km and the radial extent of the waves extends from a few kilometers to many hundreds of kilometers.

Shu et al. [4] and Borderies et al. [5] have developed the theory of nonlinear density waves. Borderies, Goldreich and Tremaine (BGT) [1] developed the theory in the presence of damping associated with interparticle collisions. This model simulates the sharp peaks and shallow troughs of Saturn's density waves. The rapid damping of strong waves is not completely understood.

The dispersion behavior of some spiral density waves in Saturn's A ring was analyzed using the non-linear dispersion relation and models described in BGT [1]. We generated dispersion profiles using the Burg power spectral technique on both the RSS and PPS data [6]. From these dispersion profiles, the values for the wavenumber k were selected at the radii of the optical depth peaks. These k values were then plotted as a function of distance from resonance.

The dispersion behavior of the Mimas 8:5 and Prometheus 7:6 density waves is linear in both the RSS and PPS occultation data. In the linear case, the BGT model produces a constant surface mass density which is in good agreement with the value we calculated using the slope of the wavenumber, k. The background surface mass density is very nearly constant for both waves. For the Mimas 8:5 density wave the average background surface mass density is $32.6 \pm 4.1$ gm/cm$^2$. This value is in good agreement with the value of $33.3 \pm 6.0$ gm/cm$^2$ calculated by Horn [6]. For the Prometheus 7:6 density wave the average background surface mass density is $42.9 \pm 2.6$ gm/cm$^2$ while the value from Horn [6] was $41.8 \pm 0.8$ gm/cm$^2$.

About 1000 km radially inward from the Encke gap in Saturn's A ring is the Mimas 5:3 density wave. The optical depth profiles of the Mimas 5:3 density wave are shown in Figures 1a
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and 1b for the PPS and RSS data. Dispersion profiles were generated and the values for the wavenumber k are selected at the radii of the optical depth peaks. These k values are then plotted as a function of distance from resonance (see Figure 1c). The dispersion behavior of the Mimas 5:3 density wave is nonlinear from the outset and remains nonlinear over the course of the wave. Using the dispersion profile we compared the results to models by Longaretti and Borderies [7] and BGT [5]. We used the nonlinear dispersion relation for a tightly wound density wave near a Lindblad resonance and calculated the background surface mass density assuming that the opacity is constant (see Figure 1d). We measure an enhancement in the background surface mass density in the wave zone outside resonance (the first two optical depth peaks). The background surface mass density is not the same in the RSS and PPS data. This inconsistency may be indicative of different processes affecting the large (RSS) and small (PPS) particle populations close to resonance. This work was supported by NASA’s Planetary Geology and Geophysics Program.

Figure 1: Mimas 5:3 Density Wave: a) Voyager PPS optical depth profile as a function of distance from Saturn. Squares indicate optical depth peaks. b) Voyager RSS optical depth profile. Squares indicate optical depth peaks. c) Wavenumber k as a function of distance from resonance location. Triangles are PPS data and +’s are RSS data. d) Surface Mass density as a function of distance from resonance. Triangles are PPS data and +’s are RSS data.