MODELLING PLANETARY CRATERS WITH ORTHOGONAL FUNCTIONS. D. Wallis, Unit for Space Science and Astrophysics, School of Physical Sciences, University of Kent at Canterbury, Kent, CT2 7NR, U.K. (dw22@ukc.ac.uk), A.T. Kearsley, Geology Dept., Oxford Brookes University, U.K., S.K. Dunkin, Dept. of Physics and Astronomy, University College London, U.K., C.J. Solomon, Applied Optics Group, School of Physical Sciences, University of Kent at Canterbury, Kent, CT2 7NR, U.K., N. McBride, Unit for Space Science and Astrophysics, School of Physical Sciences, University of Kent at Canterbury, Kent, CT2 7NR, U.K.

Impact cratering is the geologic process which has had the greatest effect on the surface of the smaller planetary bodies in our Solar System; a process that has shaped the surfaces of the Moon, Mercury, Mars, and many of the satellites. Impact craters record the later stages of planetary accretion and the continuing process of meteoroid collisions. Efforts have been made to understand details of the collision event from the crater morphology, a topic of research particularly popular in the last 20 years which has exploited the images returned from planetary missions.

Many of the methods used to classify and characterise impact crater morphology rely on comparing particular measurements of crater dimensions, such as the depth at the centre of the crater, the height of the rim, peaks, height of the central peak or rim, crater diameter etc. Craters with different sizes are often compared by dividing a measurement by the crater diameter, for example, the ratio of the crater depth to diameter is a common parameter for describing morphology. Another commonly used method is to describe the shape of the crater bowl as the function $z(r, \theta)$, where $z$ is the height, $r$ is the distance from the centre, and $\alpha$ and $b$ are found for each crater. These methods are limited. Not every aspect of the crater shape will be considered, many measurements assume that the crater has radial symmetry and none of these methods will consider every measurement on a high resolution gridded data set.

A gridded data set may contain many thousands of height measurements for a single crater. To compare crater morphologies, the measurements for each crater must be compared. Comparing every measurement is not practical, and selecting only some measurements for comparison has the limitations described above. A method[1][2] has been developed that will reduce the thousands of height measurements to a simple set of parameters and allow quantitative comparisons to be made between craters but still consider all the data points on the grid. The method can also compare radially asymmetric features of the crater.

The surface shape of an impact crater can be considered as a function, where the height $z = f(x, y)$ or $z = f(r, \theta)$. This function can be defined in terms of a series expansion

\[ z(r, \theta) = \sum_{k=1}^{N} \alpha_k \psi_k(r, \theta) \quad (1) \]

where $\alpha_k$ are the coefficients of expansion and $\psi_k$ form a complete set of functions (modes) orthogonal over a circular domain.

The basis functions $\psi_k$ are orthonormal and may be normalised by a suitable weight function $w(r, \theta)$ and are usually defined over a unit radius domain $D$. Mathematically, the orthonormality of the basis functions is defined as

\[ \int_D \psi_k(r, \theta) \psi_j(r, \theta) \, dr \, d\theta = \begin{cases} 1 & \text{if } j = k \\ 0 & \text{if } j \neq k \end{cases} \quad (2) \]

If we multiply equation (1) by $\psi_j(r, \theta)$ and integrate over $D$, then

\[ \int_D z(r, \theta) \psi_j(r, \theta) \, dr \, d\theta = \sum_{k=1}^{N} \alpha_k \int_D \psi_k \psi_j \, dr \, d\theta \quad (3) \]

By using the integral properties in equation (2), we can see that all the terms where $j \neq k$ will be zero, and the integral part of the r.h.s. of equation (3) will be 1, only when $j = k$. Equation (3) can thus be simplified to

\[ \int_D z(r, \theta) \psi_j(r, \theta) \, dr \, d\theta = \alpha_k \quad (4) \]

allowing us to calculate the coefficients for each crater using actual crater measurements for $z(r, \theta)$.

The basis functions can be any complete set of functions orthogonal over a circular domain. Two suitable choices that can be derived analytically are the Zernike polynomials and the eigenfunctions of a vibrating circular membrane. Two of the Zernike functions (4 and 39) are shown in figures 1 and 2.
The choice of the domain of integration is significant. Selecting a different domain (e.g., a small horizontal translation or radius change) will significantly influence the expansion and will produce a different set of coefficients. It is vital that a repeatable method is used to specify the domain. One solution is to choose the distance from the centre of the crater a point where the planet's surface is no longer modified (the region inside is modified by the impact, with the flat surface of the planet outside). However, this is very hard to define and the errors will be large. Another possibility is to use the crater rim peaks, but these are often found to be asymmetric. The most reliable method is to determine the edge of the crater bowl using a photograph. Although circular and easy to identify, it requires a photograph matched to the height data. This is available with the Lunar 15 Apollo topography maps. To include the crater rim, the radius is multiplied by some factor to define a radius including all areas of the crater of interest. (Obviously, the same factor must be used for all craters being compared.)

Figure 3 shows a contour map of Hadley impact crater, a good example of simple morphology for a Lunar crater. The radius used to define the domain of integration is 1.333 x bowl edge taken from the original photograph. Figure 4 shows the coefficients of expansion using the Zernike basis functions (the Zernike spectrum). The bar graph clearly shows that only a few of the terms make a significant contribution to the shape. Zernike Mode 1 is simply defined as \( z = 1 \), and makes no contribution to the shape. Mode 4 (parabolic) makes a large contribution and Mode 11, the next radially symmetric mode, makes the third biggest contribution. Small peaks can be seen for the other radially symmetric modes, and the other coefficients define the asymmetric components.

The list of expansion coefficients, the function set and the scale (the diameter for example) are all that is required to reconstruct the crater. The coefficients therefore must hold all possible information about the shape. The initial data points consisting of many thousands of measurements have been reduced to a few simple parameters without any significant loss of information. The parameters allow quantitative comparisons to be easily made. A very simple model can be made from the most significant terms that still closely matches the crater shape.

**References**

1. L. Kay, A. Podoleanu, M. Seeger and C.J. Solomon, 

2. C.J. Solomon, M. Seeger, J. Curtis and L. Kay, 