SECULAR RESONANCE SWEEPING IN A SELF–GRAVITATING PLANETESIMAL DISK, WITH APPLICATION TO THE KUIPER BELT. J. M. Hahn, *Lunar and Planetary Institute, Houston TX 77058, USA, (hahn@lpi.usra.edu)*, W. R. Ward, *Southwest Research Institute, USA, (ward@boulder.swri.edu)*.

One of the more puzzling aspects of the Kuiper Belt is the high orbital inclinations exhibited by Kuiper Belt Objects (KBOs). This swarm of distant comets have apparent inclinations of $\sim 15^\circ$ that to date have defied all simple explanations. Several distinct models have been offered as explanations for the Kuiper Belt's dynamical excitation: gravitational stirring by protoplanets that may have once roamed the outer solar system [1], scattering of KBOs out into inclined Kuiper Belt orbits by the giant planets [2], inclination–pumping due to the close passage of another star [3], and sweeping the Kuiper Belt with secular resonances as the solar nebula gas dispersed [4]. Perhaps the most promising mechanism is secular resonance sweeping since this scenario can produce the very high orbital inclinations observed in the Kuiper Belt.

A secular resonance is a site where a body's precession rate matches one of the solar system's eigenfrequencies, and a body in or near a secular resonance can achieve a very high orbital eccentricity or inclination. However the location of a secular resonance is rather sensitive to the distribution of mass throughout the solar system. Thus the dispersal of the solar nebula's gas content, which involves the removal of $\sim 99\%$ of the solar system's mass, will drive many secular resonances clear across the solar system. The consequences of sweeping secular resonances was first considered some twenty years ago when it was shown that these powerful resonances can result in very high (perhaps too high?) dynamical excitation in the terrestrial zone [5]. This mechanism has also been applied to the asteroid belt [6–9], and it is considered one of two leading causes for its excitation [10]. More recently, Nagasawa and Ida (2000) showed that nebula dispersal also causes secular resonances to sweep through the Kuiper Belt which can excite substantial eccentricities and inclinations among KBOs there.

However it should be noted that all prior studies of secular resonance sweeping treat the planetesimal disk as a sea of massless particles, whereas a real system of course has mass. This could be important since, as [11] shows, the planetesimal disk's self–attraction can allow a disturbance to propagate away from a horizontal secular resonance in the form of a spiral density wave. Similarly, a spiral bending wave might also be launched from a vertical secular resonance. Consequently, when a planetesimal disk having mass is compared to a massless disk, we expect less dynamical excitation at the resonance site and more excitation in the downstream direction due to the transport of angular momentum by spiral waves.

This phenomenon is illustrated by numerically integrating the linearized Laplace–Lagrange equations for the secular evolution of an N–body system [12]. These are time–average equations of motion that retain only those slow forcing terms that vary on precession timescales. Equivalent equations can also be derived by treating each body as a gravitating ring of fixed semimajor axis a but having an eccentricity e, inclination i, and longitudes of periapse $\tilde{w}$ and node Ω that varies over

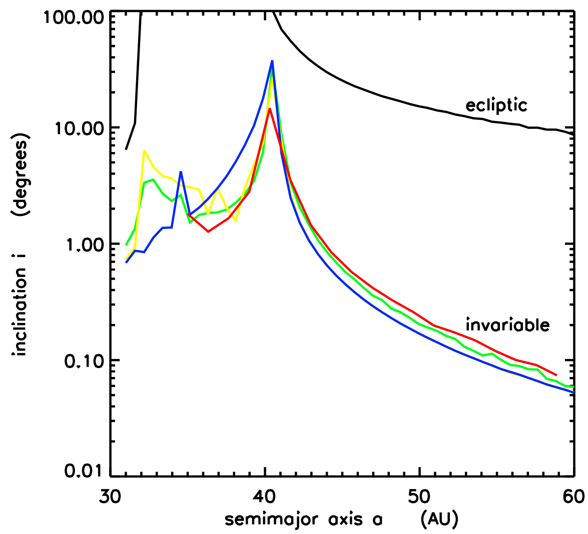

Figure 1: The mean inclination i in a massless Kuiper Belt is plotted versus semimajor axis a after dispersing the nebula gas over timescales of $\tau = 10^5$ years (green curve), $\tau = 10^6$ years (yellow), and $\tau = 10^7$ years (blue); for these runs the nebula midplane is in the invariable plane. The peak near 40 AU indicates the current location of the vertical secular resonance. Similar results are also obtained in a system having no nebula gas (red curve). The 'ecliptic' curve shows the very high inclinations that result when the nebula is instead placed in the ecliptic (which is tilted by 1.6° degrees from the invariable plane) and dispersed on a $\tau = 10^7$ year timescale.

time. Although highly idealized, this 'N–ring' integrator has two distinct advantages over a more traditional N–body simulation of the problem considered here. The first is speed, since the 10^8 years of evolution in a massless disk reported in [4] is reproduced here in only 4 hours on a 360 MHz computer. The second advantage is that the N–ring integrator can readily resolve the long–wavelength spiral waves that are launched in a gravitating disk. However evolving these systems using direct N–body methods would instead require a huge number of particles and a very large commitment of computer resources.

Our ring integrator also includes the solar nebula's time–averaged contribution to the system's disturbing function, $R_g(a) \simeq -\pi G \rho_g a^2[(h/a)e^2 + \sin i^2]$ where G is the gravitation constant, $\rho_g(a)$ is the nebula density, and $h(a)$ is its scale height [5]. The nebula's main effect is to drive rapid precession of the planets and the small–body rings, so ramping the nebula gas density ρ_g down to zero over time causes several secular resonances to sweep across the solar system.

Nagasawa and Ida (2000) effect secular resonance sweeping in a massless Kuiper Belt by placing an initial Hayashi–type minimum mass solar nebula in the ecliptic plane and letting its mass diminish to zero over an exponential timescale τ. When

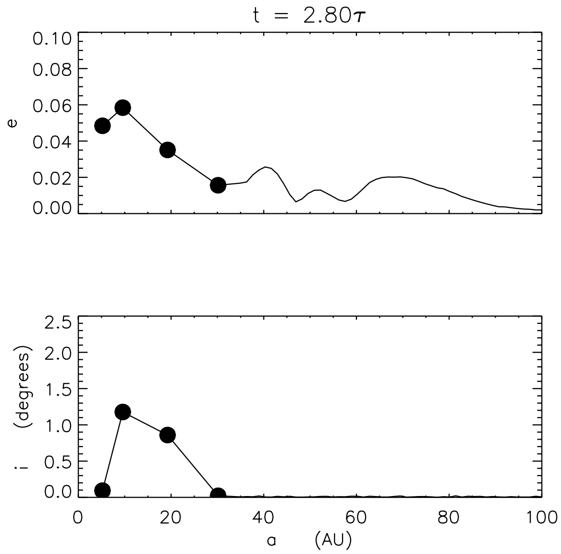

Figure 2: Click this figure to download a Quicktime animation showing the e's and i's of the four giant planets plus a 10 $M_\oplus$ Kuiper Belt as the solar nebula is dispersed over a $\tau = 10^7$ year timescale. Other file formats are available at the URL www.lpi.usra.edu/science/hahn/lpsc2002. The nebula midplane is in the invariable plane and the Kuiper Belt surface density varies as $\sigma(a) = 0.025(r/35\text{ AU})^{-3/2}$ gm/cm^3. Dots indicate the giant planets.

the nebula is slowly depleted over a timescale of $\tau = 10^7$ years, secular resonance sweeping results in modest eccentricity excitation in the Kuiper Belt (e.g., $e \sim 0.1$ at $a = 45$ AU) yet very strong inclination excitation ($i \sim 25°$ at $a = 45$ AU). Depleting the nebula over a shorter timescale generally results in a lower degree of excitation. The ring model described above has also been used to confirm all of these results.

However the high inclination excitation obtained by these models are quite sensitive to the tilt of the nebula's midplane with respect to the planets' orbits. If the nebula's midplane is placed in the solar system's invariable plane (instead of the ecliptic as was done previously), secular resonance sweeping results substantially smaller inclination excitation that is largely insensitive to the nebula depletion timescale τ (see Fig. 1). In fact the same inclination excitation results when we simply turn the system on without any nebula gas. These results demonstrate that if the solar nebula was coplanar with the current invariable plane, then secular resonance sweeping was not particularly effective at exciting inclinations in a massless Kuiper Belt.

But when sufficient mass is added to the system, a very different Kuiper Belt endstate is achieved. This is illustrated in Fig. 2 which shows an animated history of the system's e's and i's as the nebula gas is dispersed over a $\tau = 10^7$ year timescale. In this scenario, the four giant planets and 50 additional rings representing the Kuiper Belt plus a Hayashi minimum–mass solar nebula are evolved to time $t = 10\tau = 10^8$ years. This particular model has a total mass of 10 earth-masses in a Kuiper Belt that ranges over $35 < a < 100$ AU. Initially, very little happens since the nebula's gravity dominates over all other perturbations until $t \simeq 2.5\tau$ (which is when the nebula gas density is reduces by a factor $e^{-2.5} \simeq 0.08$), after which the planetary perturbations become important and KBO eccentricities are excited. But in this instance the Kuiper Belt's self–attraction allows this disturbance to propagate away in the form of a spiral density wave. Similarly, inclinations are excited at the later time $t \simeq 5\tau$ when the nebula gas is reduced by a factor ~ 0.007, and this disturbance also propagates outwards as a spiral bending wavecrest. In the absence of any wave damping, these waves propagate outwards and reflect at the Kuiper Belt's outer boundary only to return to the inner Belt and the giant planets and rebound again.

Although Figure 2 might suggest that these spiral waves ultimately loose their coherence and dissolve into random motions, higher resolution simulations using more rings still need to be performed in order to insure that this is not simply a resolution effect due to treating a Kuiper Belt with only 50 rings. Also, the N–ring integrator computes the radial forces (which give rise to density waves) between rings using a simple formula given in [12] that applies when rings are in nested, non–overlapping. Although this is probably adequate early in the simulation when the initial spiral density wavecrest is advancing through the Kuiper Belt, these radial forces might not be calculated correctly at later times when the wave has rebounded and the rings appear disordered and in marginally overlapping orbits. Although these issues will be addressed as we develop the next generation of this N–ring integrator, they are not expected to alter our primary findings given below.

Other systems where the total Kuiper Belt mass is 1, 10, and 100 $M_\oplus$ have also been evolved with nebula depletion timescales of $\tau = 10^5$, 10^6, and 10^7 years, and models having total Kuiper Belt masses exceeding ~ 1 $M_\oplus$ all give rise to Kuiper Belt endstates that are qualitatively similar to that seen in Fig. 2. In these instances the Kuiper Belt histories are largely insensitive to the nebula depletion timescale τ. In general, when the total Kuiper Belt mass exceeds ~ 1 $M_\oplus$ over $35 < a < 100$ AU (*i.e.*, when the KBO surface density exceeds $\sigma \gtrsim 0.003$ gm/cm^3 at $a = 35$ AU, which is $\sim 10\times$ its *current, depleted* surface density) during the epoch of nebula dispersal, long wavelength spiral waves redistributes the giant planets' disturbances throughout the Kuiper Belt. As Fig. 2 suggests, the propagation of spiral waves launched by the giant planets in the Kuiper Belt substantially reduces the degree of excitation seen in the inner $a \lesssim 40$ portion of the Belt, but still results in low–level excitation that spans the entire Belt.

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