IMPACT TSUNAMI CALCULATIONS: HYDRODYNAMICAL SIMULATIONS VS. LINEAR THEORY. D. G. Korycansky, E. Asphaug, S. N. Ward, *CODEP, Department of Earth Sciences, University of California, Santa Cruz CA 95064 USA (kory@es.ucsc.edu).*

Tsunamis generated by the impacts of asteroids and comets into the Earth's oceans are widely recognized as a potential catastrophic hazard to the Earth's population (e.g. Chapman and Morrison 1994, *Nature* **367**, 33, Hills *et al.* 1994 *Hazards Due to Comets and Asteroids*, ed. T. Gehrels, 779, Atkinson 2000, *Rept. UK Task Force on Potentially Hazardous NEOs*, http://www.nearearthobjects.co.uk). A number of hydrodynamic simulations of tsunami impacts have been carried out (recently for example Gisler *et al.* 2003, *Sci. Tsunami Hazards*, **21**, 119). While 2D and 3D hydrodynamical simulations such as those of Gisler *et al.* are extremely impressive, they are also highly computationally intensive; it is especially difficult if not infeasible to track the tsunami waves over ocean-basin scales (thousands of km), due to the inordinately large computational grid required to resolve waves whose amplitude decreases approximately $\propto 1/r$ from the impact point. In addition, after the initial impact phase in which the impactor strikes the ocean at a highly supersonic velocity, the phenomena are governed by gravity-wave speeds $\sim (gh)^{1/2} \ll c_s$, so that the typical explicit hydrodynamic sound-speed Courant timestep limit is unnecessarily restrictive. At this stage classical water-wave theory becomes useful.

Ward and Asphaug have applied linear water-wave theory to tsunami wave propagation (cf. Ward and Asphaug 2000, *Icarus*, **145**, 64, Ward and Asphaug 2002, *Deep Sea Res. Part II*, **46**, 1073). In these calculations, a water-cavity profile that is appropriate to an impact is taken as a starting point, and the resulting waves are evolved (or more precisely evaluated from integrals over the the wave spectrum) for later times (minutes to hours) according to linear theory for waves in an ocean of general depth. This formulation is flexible, adaptable to a wide variety of situations, and rapidly and easily computable. In addition there are no sources of poorly-constrained error such as numerical viscosity, an important point for modeling phenomena having intrinsically low dissipation on long timescales. On the other hand, linear theory is formally limited to waves of infinitesimal size, so that the product ak of amplitude a and wavenumber $k = 2\pi/L$ must be "sufficiently" small, i.e. $ak \ll 1$ for reasonable results. It is not obvious that waves of (say) 0.5 km amplitude and 10 km wavelength (which might be the largest and longest waves resulting from the impact of a 300 m diameter body and for which $ak \sim 1/3$) will be adequately described by linear theory. Thus we have carried out a number of hydrodynamic calculations of the collapse of, and wave generation from, plausible km-scale cavities in a uniform ocean of 5 km depth in order to check the validity of the application to this problem of linear theory and the limitations thereof. In addition, simple impact calculations were carried out in order to determine the optimal starting point post-impact for a linearized treatment.

The hydrodynamical calculations were done using the code ZEUS3D (Stone and Norman 1992, *Ap. J. Supp.* **80**, 753) as modified by the inclusion of tracer variables for multi-material calculations (Mac Low and Zahnle 1994, *Ap. J.* **434**, L33, Zahnle and Mac Low 1996, *Icarus*, **108**, 1). The tracer variables track the advection of water (and/or rock for an impact calculation) on the numerical grid; no other interface treatment (such as marker particles) is used. If the grid resolution is high enough, diffusion of tracer is minimal and the atmosphere-ocean interface is adequately sharp. Zones for which the tracer is present have their pressures set according to the appropriate equation of state (water or rock), for which we use the Tillotson formula (Melosh 1989, *Impact Cratering*). ZEUS3D is capable of giving good results for incompressible fluid flows such as growth of the classical Rayleigh-Taylor instability (Jun *et al.* 1995, *Ap. J* **453**, 332, Korycansky *et al.* 2002, *Icarus*, **157**, 1). The calculations described here were 2D and axisymmetric with a rigid reflecting bottom and outer boundary at 150 km radius, to afford the simplest and most precise comparison with linear waves. In order to ensure a hydrodynamically "quiet start", a pressure gradient to balance gravity was included in undisturbed portions of the ocean, along with a corresponding density profile in order to avoid convective instability due to unstable entropy gradients. We tested grid sizes up to 520×2000 were used for regions of size 10 km or 13 km in vertical extent (of which 5 km was the ocean $z < 0$) and 50 km or 150 km in radial extent. In some cases, non-uniform grids with geometrical expansion factors (typically 1.04× per zone) were used as a test of the possibility of the improved computational efficiency of such grids. Arbitrary waveforms are possible for initial conditions; a typical initial wave profile $\zeta(r)$ was given by a parabolic shape for $r \le 1.5r_0$ and a $1/r^3$ lip for $r \ge 1.5r_0$:

$$\begin{aligned} \zeta &= h\left[1+q\left(\frac{r^2}{r_0^2}-1\right)\right], \quad r \le 1.5r_0, \\ \zeta &= h\left[1+\left(\frac{5q}{4}\right)\left(\frac{1.5r_0}{r}\right)^3\right], \quad r \ge 1.5r_0, \end{aligned} \tag{1}$$

where h is the ocean depth (5 km), q is the relative depth of the cavity ($q = 0.4$ usually), and r_0 is the cavity radius (5 km).

For linear theory various formulations are possible. Here we use the simple equations of potential theory for the surface $\zeta(r)$ and velocity potential $\phi(r,z)$ (Drazin and Reid 1981, *Hydrodynamic Stability*):

$$\frac{\partial \zeta}{\partial t} = \frac{\partial \phi}{\partial z}, \quad \frac{\partial \phi}{\partial t} = -g\zeta \tag{2}$$

which is satisfied for axisymmetric waves in a cylindrical basin with rigid bottom and outer boundary at radius R by

$$\begin{aligned} \zeta(r,t) &= \sum_n a_n^{\zeta}(t) J_0(k_n r), \\ \phi(r,z,t) &= \sum_n a_n^{\phi}(t) J_0(k_n r)\cosh[k_n(z+h)], \end{aligned} \tag{3}$$

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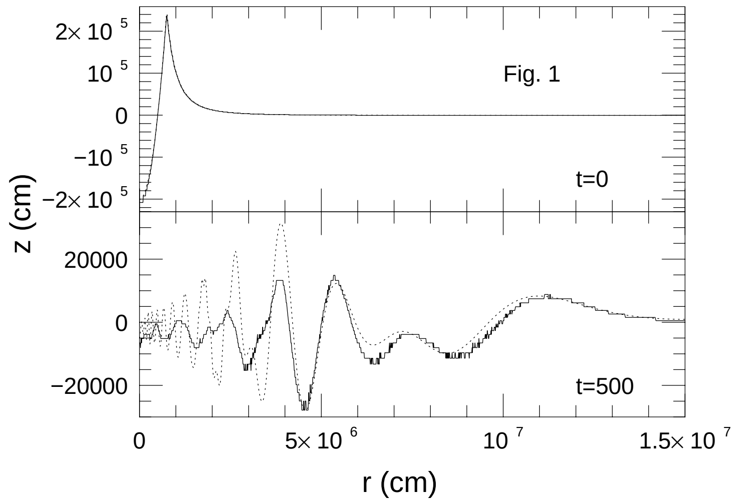

where the mode wavenumbers k_n are fixed by the boundary condition $\partial\phi/\partial r = 0$ to satisfy $J_1(k_n R) = 0$. The mode amplitudes $a_n^\zeta(t)$ and $a_n^\phi(t)$ are given by

$$a_n^\zeta(t) = A_n \cos\omega_n t + B_n \sin\omega_n t$$
$$a_n^\phi(t) = \frac{\omega_n}{g}\left(A_n \sin\omega_n t - B_n \cos\omega_n t\right), \quad (4)$$

where A_n and B_n are set by the initial conditions ζ and ϕ at $t = 0$: $A_n = a_n^\phi(0)$, $B_n = -g a_n^\zeta(0)/\omega_n$. The mode frequency ω_n is given by the well-known dispersion relation for waves in an ocean of depth h: $\omega_n^2 = g k_n \tanh(k_n h)$. The mode amplitudes are found by the (crude) method of a least-squares fit of mode amplitudes from Eqn 3 to the initial profile of ζ at $t = 0$. For simplicity we take a case for which the initial profile is at rest so that $\phi(r,z,0) = 0$ and thus the velocity field (for the hydrocode) $v = \nabla\phi = 0$.

A fairly typical example of a hydrocode vs. linear theory is shown in Fig. 1, for the (parabola+$1/r^3$ lip) profile given above. The two panels show the initial ζ profile and the ocean surface at the end of the calculation at $t = 500$ sec. (Note the difference in vertical scale between the two panels.) The ZEUS3D grid was $z, r = 130 \times 1500$ in size, non-uniform in z and uniform in z; for the linear theory calculation there were 400 radial modes. As predicted by wave theory, the waves disperse so that longest waves propagate fastest and become the leading waves that would arrive on shore first. Agreement between linear and non-linear calculations is good for the first 6 leading peaks and troughs ($r > 5 \times 10^6$ cm). In particular, there is no reason to think that wave amplitudes drop more rapidly than $\sim 1/r$ as predicted by linear theory. Interior to $r = 5 \times 10^6$ cm, where linear theory shows modes of increasingly large wavenumber, disagreement between linear and non-linear formulation is significant. Large-k modes are expected to become non-linear more quickly due to the larger gradients involved (assuming a constant amplitude). The non-linear calculations generally show lower amplitudes in the large k regime; while this is likely due to non-linearity and mode-mixing, it is also possible that numerical viscosity spuriously damps short wavelengths. (The initial collapse of the cavity produces a transient peak at $r = 0$, $t \sim 40$ sec, which non-linear simulations show as being much larger than predicted by a linear model. However, it appears that the transient peak has little influence on the leading waves that propagate most rapidly to large radii.) An important consideration is the wave spectrum corresponding to the initial conditions. In Fig. 1 the slope discontinuity at the lip of the cavity ensures that there is a relatively large amount of power at large values of k; tests with smooth initial conditions produce less amplitude in short waves and much closer agreement between linear and non-linear calculations at small radii.

A pertinent aspect for the issue of linear-theory predictions is the choice of initial condition. It is possible to specify the initial surface profile and one component of the surface velocity (assuming potential theory). These can be applied at some optimal time after the impacts, e.g. the moment when the kinetic energy is at a minimum and the water cavity most resembles the initial condition above (where the initial kinetic energy was zero). Further study is necessary to identify this optimal time point.

Our general conclusion is that linear theory is a reasonably accurate guide to behavior of tsunamis generated by impactors of moderate size, where the initial transient impact cavity is of moderate depth compared to the ocean depth. This is particularly the case for long wavelength waves that propagate fastest and would reach coastlines first. Such tsunamis would be generated in the open ocean by impactors of ~ 300 meters in diameter, which might be expected to strike the Earth once every few thousand years, on the average. Larger impactors produce cavities deep enough to reach the ocean floor; even here, linear theory is applicable if the starting point is chosen at a later phase in the calculation when the impact crater has slumped back to produce a cavity of moderate depth and slope.

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