A SOLAR SYSTEM SURVEY OF FORCED LIBRATIONS IN LONGITUDE. Comstock, R.L.[1], Bills, B.G.[2,1], *1. Scripps Institution of Oceanography, La Jolla CA 92037, USA*, *2. NASA Goddard Space Flight Center, Greenbelt MD 20771, USA*.

Introduction: Physical librations in longitude are forced periodic variations of a body's rotation rate. If the torque producing the librations can be calculated, then observations of the phase and amplitude of librations can provide information on mass distribution, and effective strength of the body. In the near future prospects for observing physical librations look quite promising. Radio interferometric observations of Venus and Mercury [1] may yield sufficiently accurate rotational observations that librations there may be visible. Range measurements from Earth to networks of landed instrument packages on Mars [2] are likely to yield librational data there as well. We compute expected libration amplitudes from physical and orbital parameters of a set of planets and satellites partially motivated by a desire to identify candidates for future observations.

Solar system bodies occupy one of three general rotation states: non-resonant states, resonant states, and the synchronous resonant state. Analytical treatments of forced librations were initially motivated by the Moon. Lunar librations were predicted by Newton, first detected telescopically by Bessel [3], and definitively resolved through lunar laser ranging [4] which has led to quite thorough analysis of of librations for the synchronous case[5, 6, 7, 8]. The synchronous resonant state is commonly observed among satellites. The only known body to exist in a non-synchronous resonance is Mercury [9] which exists in a 3:2 resonance, completing three rotations for every two revolutions about the sun. The analysis of Goldreich and Peale [10] has lead to improved understanding of the general case of half integer resonance states. The dynamics of forced librations in non-resonant rotators has received less attention. While there are few cases in which non-resonant forced librations have been observed, Earth is an important exception, and current observing techniques may have the capacity to detect them on Venus.

A comprehensive observing program spanning a range of solar system bodies can address an array of geophysical issues involving interior mass distribution of planets, satellites, and asteroids. Calculations of expected librations can supply amplitude estimates helpful in identifying the likelihood of detecting librations observationally. We present a survey of expected libration amplitudes for a subset of solar system bodies, identifying those bodies with amplitudes likely to be detectable, and commenting on spin state, and radial structure implications.

Theory: The equation of librational motion in the absence of tidal torques can be written as

$$\frac{d^2\theta}{dt^2} = -\frac{\omega^2}{2}\left(\frac{a}{r}\right)^3 \sin\left[2(\theta - f)\right] \qquad (1)$$

where a is the semi-major axis, r is the radial distance from the rotator to the torquing body, and f is the orbital true anomaly. The orientation of the axis of least inertia with respect to an inertial reference frame is specified by the angle θ between the axis and the reference direction, and ω is the frequency of free librations

$$\omega^2 = 3n^2\left(\frac{B-A}{C}\right) \qquad (2)$$

where n is the body's mean motion, and its principal moments of inertia are $A \leq B < C$.

Equation (1) requires an estimate of the moment difference $(B-A)/C$ to define ω, the frequency of free librations. For bodies where knowledge of this parameter is incomplete we assume homogeneity and apply either a hydrostatic equilibrium approximation (appropriate for the larger outer satellites), or a shape model (appropriate for smaller rigid bodies such as Phobos) to construct an inertia tensor from which a plausible ω can be estimated.

While equation (1) is geometrically convenient, the time evolution of the orbital track in r and f is non-linear with non-zero eccentricity. It will prove convenient to express orbital position as a function of mean anomaly M, a quantity that increases linearly with time. If we assume nearly steady rotation then the angle θ can be decomposed into the small libration angle γ and its steady state rotation angle pM

$$\theta = pM + \gamma \qquad (3)$$

where p is the ratio of the orbital period to the rotation period, a statement of the relationship between orbital, and rotational motion. On the assumption that the libration angle γ is small enough to be neglected in calculating the torque, it can be shown that the libration amplitude can be written as [11]

$$\gamma(t) = \left(\frac{\omega}{2n}\right)^2 \sum_{q=1}^{\infty} G_q^+ \sin[(2p+q)nt] + G_q^- \sin[(2p-q)nt]. \qquad (4)$$

The coefficients G_q^+, and G_q^- are functions of eccentricity e, p, and q, and include Fourier series expansions of the forcing function generated through application of the appropriate Cayley expansions [12]. Thus the libration angle γ can be written as a sum of oscillations, each of which is periodic.

Note that the frequencies are all related to the semi-diurnal sidereal rate $(2pn)$, and include side-bands which are evenly spaced above and below it, in integer steps of one cycle per orbit. For a rapid rotator, like or Earth or Mars, these frequencies will all be close to the main semi-diurnal frequency. For a slow rotator, like Venus, some of these side bands will be quite far from semi-diurnal. Inspection of the coefficients shows that the largest will be the term G_2^-, which is associated with the semi-diurnal forcing, as seen from the torquing body, at frequency $2(p-1)$.

Application: The analysis developed above makes no a priori assumption of resonance, allowing for analytical treatment of non-resonant rotators, and makes libration analyses possible for a broader range of planetary bodies. However, we have assumed principal axis rotation, and zero obliquity, thus for bodies with finite obliquity, such as Earth and Mars, there are components of gravitational torque that will excite latitudinal librations, presenting additional complexities. As the librations in most bodies are expected to be small, the latitudinal and longitudinal perturbations are nearly decoupled, and we defer analysis of latitudinal effects for the present.

A primary motivation for examining forced librations is the expectation that observations of them can yield information about the moments of inertia of the rotating body. Tracking the trajectories of natural or artificial satellites in orbit about a planet can yield information about the low degree gravity field [13]. The relationship between the five degree two spherical harmonic coefficients, and the moments and products of inertia are insufficient to completely populate the inertia tensor. Observations of physical librations provide a method for completely defining the inertia tensor by yielding estimates of the moment difference $(B-A)/C$. Comparison of theoretical libration amplitudes with observed amplitudes can provide insight into interior density structure.

As an example we apply the analysis to Venus, a slow, non-resonant rotator ($p \simeq -12.02/13$). Using the five degree two spherical harmonic gravity coefficients, and an estimate of the mean moment of inertia $J \simeq 1/3MR^2$ [14] we produce an estimate for the moment difference $(B-A)/C$, with associated free libration frequency $\omega/n = 4.47 \times 10^{-3}$. Inserting eccentricity e, p, ω, and n into the formulas for libration amplitude, we obtain for the dominant semi-diurnal term $G_2^- = -6.737 \times 10^{-7}$. That angular amplitude corresponds to a displacement of a point on the equator of Venus by only 4.1 m over the course of the libration. The period of the forced libration is $2\pi/[2(p-1)n] = 58.376$ day. During the course of a librational cycle, the instantaneous rotation period will vary over the range 243.019 day $\leq |2\pi/\Omega(t)| \leq$ 243.021 day.

Figure 1 illustrates the spectrum of librational motions, using the formulation shown above. The term of largest amplitude is at two cycles per mean solar day on Venus. Other shown frequencies are offset from that by integer numbers of cycles per orbit. Since the year and day on Venus are comparable in length, the side-bands are well separated from the main line. Note that the amplitudes of the sidebands generally decrease with increasing order q, which is a result of having the leading order terms involve higher powers of the orbital eccentricity. An exception, for Venus, is the G_2^+ term, with frequency $2(p+1)n$. Due to the near commensurability between spin and orbit rates for Venus ($p \simeq -12/13$), this term has a very low frequency (4.08 year period) and correspondingly large amplification in response to the the torque.

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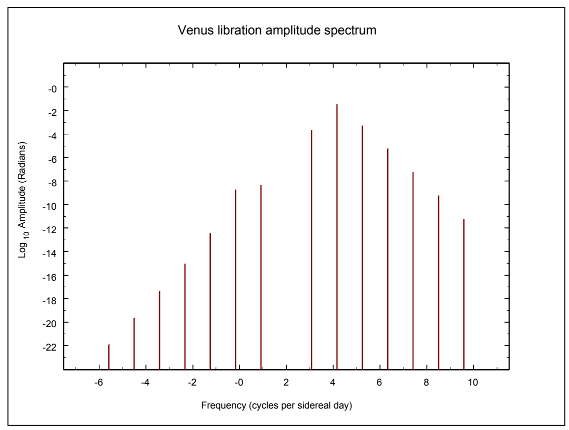

Figure 1: Forced libration amplitude spectrum for Venus. Each line represents the amplitude in radians and frequency in cycles per sidereal day of a component of forced libration. A microradian of angular displacement on Venus corresponds to a distance of 6.05 m at the equator, and the durations of the sidereal and mean solar days are 243.02 and 116.76 Earth days, respectively.