SPATIAL AND TEMPORAL PATTERNS OF TIDAL DISSIPATION IN SYNCHRONOUS SATELLITES

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Introduction: Tidal heating is an important energy source for several solar system bodies, and there is a wide-spread perception that the pattern of surface heat flow is diagnostic of internal structure. We wish to clarify that situation.

Our analysis depends upon two important assumptions: First, that heat transport is dominated by conduction. Second, that the body can be modeled by a sequence of spherically symmetric layers, each with a linear visco-elastic rheology. Under these assumptions, surface heat flow patterns in tidally dominated satellites will reflect radially integrated dissipation patterns. For synchronously rotating satellites with zero obliquity, this pattern depends quite strongly on orbital eccentricity but relatively little on purely radial variations in internal structure. The total amount of heat generated within the body does depend sensitively on internal structure, but the spatial pattern is rather insensitive to structure, especially at low orbital eccentricities.

Finite obliquity can complicate the spatial and temporal pattern of dissipation. Motion of the tidal bulge north and south of the equator causes an additional strain field which interacts with the patterns due to radial and longitudinal tidal components. The radial and longitudinal patterns repeat once per anomalistic cycle and the latitudinal pattern follows the nodical cycle. As the nodal line advances and the apsidal line regresses, these tidal components form changing patterns of dissipation. Though tidal evolution will tend to damp initially large obliquity values, they are not expected to vanish entirely, and thus temporal variations in tidal heat generation might be important.

Calculation Strategies: Calculating the rate of tidal dissipation for a specified internal structure model can be done in at least two different ways. The clearest approach, conceptually, is to calculate stress, strain, and strain rate at each point within the body, and average the product of stress and strain-rate over the period of tidal forcing [1,2]. That averaged product yields the volumetric dissipation rate. This approach is very general, and allows radial dissection of the dissipation pattern. However, the expressions obtained can be quite complex in form, and the global properties are not evident by inspection.

If the radial integral of this local dissipation rate is sought, an alternative approach [3,4] is to take the product of the imposed tidal potential and the time derivative of the induced potential, due to deformation, and average that quantity over the forcing period. With appropriate scaling, this potential product also yields the expected surface heat flow and is much easier to implement. The principal advantage of this approach, in the current context, is that it makes the dependence on internal structure much easier to see.

The degree 2 tidal potentials, both imposed and induced, can each be written as a sum of 3 spherical harmonic basis functions. The dissipation pattern, which is constructed from the time average of the product of the potentials, has 9 separate terms in the initial product, but only 4 of them survive the time averaging. Each of the surviving terms can be written as a sum of spherical harmonics of even degree l (0, 2, or 4), even order m (0, 2, or 4), and even parity (cosine in longitude). The resulting dissipation pattern has 3 orthogonal planes of reflection symmetry, with the equator and prime meridian delineating two of them.

Spatial Pattern and Internal Structure:

The expected pattern of surface heat flow from tidal dissipation, for the case of zero obliquity, can be written as

$$W(\theta,\phi) = V\sum_{i=1}^{4} F_i(\theta,\phi)\sum_{p=1}^{\infty} p\,\mathrm{Im}[k_2(pn)]H_i(p,e)$$

where the dimensional multiplier is $V = 5\,n^2 R^3 / 4\pi G$, and θ is latitude, φ is longitude, n is orbital mean motion, e is orbital eccentricity, R is the satellite mean radius, G is the gravitational constant, and k_2 is the tidal Love number, which parameterizes the resistance to tidal deformation. The spatial pattern is represented by the four basis functions F_i, each of which is a sum of spherical harmonics of even degree and even order. The temporal variations in potential are represented by the functions H_i, which are functions of eccentricity and frequency.

Written in this form, it may still not be evident that the dissipation pattern can be factored into a product of a global value, which depends on eccentricity and internal structure, and a spatial template, which depends on eccentricity, but is very nearly independent of structure. To make that factorization evident, we must examine the relationship between internal structure and the form of the tidal Love number.

In the Laplace transform domain, the imaginary part of the complex Love number for a radially stratified Maxwell visco-elastic body can be written in the form [5]

$$\mathrm{Im}[\widetilde{k}_2(\omega)] = \sum_{j=1}^{N} \frac{r_j\,\omega}{s_j^2 + \omega^2}$$

where ω is the forcing frequency and the poles s_j and residues r_j depend on the internal structure. For a homogeneous incompressible body there is a single relaxation mode, with associated pole and residue. Each density jump in the interior will introduce an additional buoyancy mode, and each jump in Maxwell relaxation time (change in either viscosity or rigidity) will introduce two additional transition modes [6, 7]. Thus, for a general L-layered body, one would anticipate a total of $N = 3L - 2$ modes.

The key observation is that each of the visco-elastic relaxation modes makes a formally identical contribution to the dissipation pattern. If the temporal factors in the tidal potential are truncated at the first order in orbital eccentricity then the normalized heat flux, defined as the ratio of heat flux at a surface point divided by the global average value, will be the same for a homogeneous body, with one relaxation mode, as for a body with arbitrarily complicated radial structure. Each of the relaxation modes contributes identically to the normalized pattern.

When higher order terms in the Taylor expansion in eccentricity are included, as would be necessary for higher eccentricity orbits, the independence of dissipation pattern on structure is destroyed. However, even for reasonably high eccentricity values, the dependence on structure is still rather slight.

Implications: This issue might seem to be entirely academic except for the fact that there is a widespread perception that the pattern of dissipation depends on internal structure. If that were true, then observations of heat flux on Io, for example, might allow discrimination between various proposed internal viscosity patterns.

The perception that tidal dissipation patterns are sensitively dependent on internal structure appears to have originated from a series of important papers [8,9,10] in which various aspects of dissipation within Io were being explored. Figures 8 and 10 of [9] have been frequently used as end-member patterns for comparison to observations of inferred heat flow from Io [11,12]. Figure 8 is for a homogeneous model, and it yields maxima at the poles and minima at the sub- and anti-Jovian points. Figure 10 is for a model in which dissipation mainly occurs in a shallow asthenosphere. It has maxima near the equator and vanishing heat flow at the poles.

Limitations: The arguments presented above are strictly applicable only to bodies in which the material properties (density, rigidity, viscosity) are functions of radius alone. Any lateral variations in material properties will invalidate the invariance of the dissipation pattern. For instance, tidal dissipation in the Earth, which mainly occurs in the oceans, exhibits a complex pattern, as it is a strong function of ocean depth and density stratification [13]. Furthermore, dissipation is assumed to occur in a material that can be modeled with a linear visco-elastic rheology. Strictly, conduction is assumed to dominate heat transport within the body, neglecting the effect of a convecting molten layer substantially altering the dissipation pattern [9,14].

Temporal Variations: If the obliquity of a synchronous rotator is non-zero, there will be relatively long period variations in tidal dissipation rate and pattern. The obliquity of Io is certainly small, but likely non-zero. Assuming that tidal dissipation has driven it to a Cassini state, the obliquity can be estimated to be $4.1\ 10^{-5}$ radian. [15]. In addition to the short period tides associated with the orbital period, there is also a family of long period tides whose dominant period is set by the relative motions of the apsidal and nodal lines. This period is 3.35 years for Io [16]. It is interesting to note that Loki, the largest volcano of Io appears to have periodic heat output with a period of 540 days [17], very nearly ½ of this tidal period, as would be expected if tidal effects were driving that variation.

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