N-BODY SIMULATION OF THE URANIAN ECCENTRIC RINGS. H. Daisaka, *University of Tokyo, Tokyo 113-0033, Japan, (daisaka@astron.s.u-tokyo.ac.jp)*, J. Makino, *University of Tokyo, Tokyo 113-0033, Japan*.

Introduction: One of the most striking features of the Uranian ring system is that some of rings, at least 6, 5, 4, α, β, ϵ-rings, are eccentric (e.g., reviewed by French et al. [1]). The existence of the eccentric rings is truly surprising, because it requires the pericenter of ring particles with different orbital periods to behave in exactly the same way under dispersive effects such as quadrapole moment of Uranus, which precesses the longitude of the pericenter of a ring particle, ϖ, depending on its semimajor axis, a. If the eccentricity, e, and inclination, i, are small enough ($e, i << 1$), the precession rate is expressed as

$$\frac{d\varpi}{dt} \simeq \frac{3}{2} J_2 R^2 (GM)^{1/2} a^{-7/2}, \qquad (1)$$

where G is the gravitational constant, and, M, R, and J_2 are the mass, radius, and non-dimensional harmonic coefficient of the potential of Uranus. For the ϵ-ring, the difference between the angles of the pericenter of the innermost particle and that of the outermost particle undergoes one full rotation in about 200 years. Thus, it seems unlikely that the eccentric ring survives for a long time without an additional contribution.

In order to explain these eccentric rings, various theorical models have been proposed. One is a theory based on the self-gravity of the ring, originally proposed by Goldreich&Tremaine[2,3] (also, see Borderies et al.[4]). In this model, the self-gravity of ring particles locks their apsides against the differential precession. This model is widely accepted, since the model can give some observable prediction which is consistent with observations. However, it also has consequences which contradict with observations (e.g., [1, 5]). Recently, in order to solve such problems, this model was extended to take into account the effect of particle collision, and predicted much larger mass for the α-ring [6, 7]. Kozai[8, 9] proposed another model, in which eccentric rings are maintained by the forced oscillation of ring particles caused by undiscovered shepherding satellites in eccentric orbits. In this model, it is assumed that apsidal motions of the ring particles and the satellites completely synchronize with each other. It has been unclear whether or not any of these theories correctly accounts for the formation and survival of the eccentric rings.

We studied the evolution of a ring-satellite system in which a narrow ring is confined by two shepherding satellites orbiting around an oblate central planet, by performing direct N-body simulations. This is a setting similar to Kozai's model, but we have not made any assumption on the eccentricity of shepherding satellites. In our model, the orbits of satellites evolve through interaction with the ring and the other satellite. In our simulations, gravitational interaction and physical (inelastic) collision between ring particles are taken into account. For calculation of the gravitational interactions, we used GRAPE-6 [10], which allow us to do simulations for a long time with a large number of particles.

Simulation Method and Model: We consider motion of particles under a potential of a oblate central body, taking into account mutual interactions of particles through the gravitational force and the direct collision. The orbit of particles is calculated by integrating the equation of motion with the forth-order P(EC)^n Hermite scheme [11]. For calculation of the gravitational force between ring particles, GRAPE-6, a special purpose computer for calculating gravitational force [10], is used. In our simulation, collision is detected as overlapping of particles. By assuming a free-slip, hard-sphere collision model, we calculate post-collisional velocity with restitution coefficient in normal direction, ε. For simplicity, we use velocity-independent restitution coefficient and $\varepsilon = 0.01$, which is needed to keep ring particles within a narrow region. For details, see Daisaka&Makino[12].

We consider a simple ring-satellite model in which a ring with the width, W, at the semimajor axis, a, consists of a swarm of identical particles (mass m_p and radius r_p) and the masses of the ring, m_r, and a satellite, m_s, are the same. Also, we set initial separations from the ring to inner and outer satellites to be the same distance. In this case, we set $m_r/M = m_s/M = 10^{-6}$ and $W/a = 0.01$, and $J_2(R/a)^2 = 8.95 \times 10^{-4}$. We also set distance between ring and a satellite to be $5h_s a$, where $h_s = (m_s/3M)^{1/3}$ is the reduced Hill's radius. Initial orbits of the satellites are circular but ring particles has non-zero but very small eccentricity and inclination. Angular variables are chosen to be random. Note that the satellites we consider in our model are much closer that those around ϵ-ring (typically, $> 50h_s$).

Formation of eccentric rings: Figure 1 is the result of a simulation with 10000 identical ring particles, showing the time evolution of the distribution of ring particles in cylindrical coordinates. Initially, the ring is circular, and the particles form a straight line in this coordinate (Fig.1(a)). In an earlier stage at $t = 800$ (Fig.1(b)), where T_K is the Keplerian time at a, the ring still remains circular, although the satellites creat wake structure downstream. However, at $t = 3000T_K$ (Fig.1(c)), the ring shows the sinusoidal distortion of the distribution; this means that the ring becomes eccentric. In the eccentric ring, the pericenter of ring particles tend to be aligned: Figure 2(a) shows that the distribution of particles is off the origin in the eccentricity vectors. After the eccentric ring develops, the ring remains eccentric (Fig.1(d)), although its pericenter precesses. The rate of the precession of the ring, as well as the satellites, can be roughly explained by the effect of J_2. Thus, the apsidal motion of the ring and satellites does not synchronize with each other, as Kozai[8, 9] expected.

Figure 1 also shows that a configuration of the simulated eccentric ring changes with time. The width of the ring in Fig. 1(c) is the same along the azimuthal direction, and the eccentricity of particles does not depend on the semi-major axis in this case. On the other hand, Fig. 1(d) shows that the width is narrower at the pericenter than at the apocenter, in which outer particles have a larger eccentricity. This configuration resembles that of the Uranian α, β, and ϵ rings. Such a

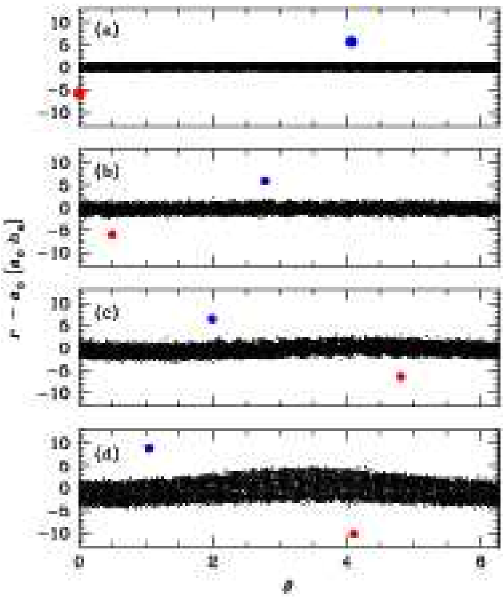

Figure 1: Formation and evolution of an elliptical ring in cylindrical coordinates. (a)$t = 0T_K$. (b)$t = 800T_K$. (c)$t = 3000T_K$. (d) $t = 19735T_K$.

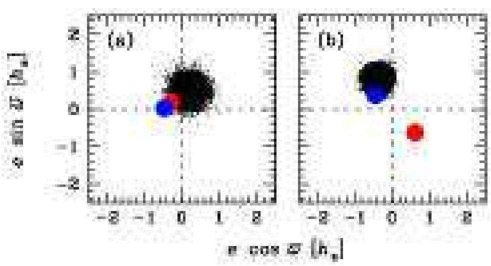

Figure 2: Particle distribution in the eccentricity vectors. (a) case at $t = 3000T_K$ in Fig. 1. (b) case where the self-gravity of ring particles is neglected. $t = 4138T_k$.

variation of the width is roughly periodic, and the period seems to be related with the synodic period of the apsidal motion of the ring and satellites.

In order to clarify the mechanism of the formation of our simulated eccentric ring, we performed a simulation without the self-gravitational force of ring particles but with the same parameters as those used in the simulation of Fig. 1. The result of the distribution of particles in eccentricity vectors is seen in Fig.2(b). The distribution is offset from the center, indicating that the ring becomes eccentric. Also, this ring remains eccentric for long time. This result indicates that the self-gravity of the ring particles does not play an important role on the formation and maintenance of our eccentric ring.

Discussion: Our *N*-body simulation demonstrated the formation and evolution of an eccentric ring from an initially circular ring confined by satellites inside and outside of the ring. The formation of the eccentric ring occurred even in the simulation without the self-gravity of ring particles. We have not fully understood the mechanism which maintains the eccentricity and the common apsidal motion of ring particles. Most likely, the secular perturbation from eccentric satellites are involved, since we found that if we fix the orbits of satellites, the ring becomes eccentric only when either satellite is eccentric [13].

In our simulation, a ring with shepherding satellites 5 h_s away from each side of the ring can develop into an eccentric ring. If we apply this relationship between the ring and satellites to the Uranian ring system, we can expect that there are undiscovered shepherding satellites about 5 h_s away from the edges of the α and β rings. Even in the ϵ-ring system, there is a possibility that in addition to the discovered satellites, Ophelia and Cordelia, shepherding satellites exist much closer to the ring than these known satellites, as supposed by Kozai[8, 9]. A unique nature of our model is that it predicts the time variation in the eccentricity of the ring and its radial gradient. If the mechanism that maintains the Uranian eccentric rings is the same as what we found in our numerical simulations, the Uranian rings should show the time variations in the width of the ring.

The figures shown here have low resolution. The original, high resolution figures can be found under following URL: http://grape.astron.s.u-tokyo.ac.jp/~daisaka/figures

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