ROLE OF THE MANNING COEFFICIENT ON PROGRESSIVE INUNDATION BY A MEGAFLOOD. H. Miyamoto[1,2] and V. R. Baker[3], [1]*Lunar and Planetary Laboratory, University of Arizona, USA, (hirdy@lpl.arizona.edu),* [2] *Department of Geosystem Engineering, University of Tokyo, Japan,* [3] *Department Hydrology and Water Resources, University of Arizona, USA.*

Introduction: Estimations of peak discharge rates and durations of floods will play critical roles in discussions about megafloods and their relevance to the history of water on Earth and Mars. However, the movement of the floodwater is not easily modeled in an exact form because it strongly correlates to turbulent forces. In addition to that, actual floods progressively inundate complicated rough terrains, which introduces an additional complexity on the movement of the megaflood. For these reasons, empirical equations, especially the Manning equation developed in the fluvial hydrology field, are widely used to analyze megafloods [e.g., 1-3]. Nevertheless, the Manning resistant coefficient of a megaflood is not easily estimated by extrapolation from measured stream flows of much smaller floods. Here we discuss its theoretical background and its role in megaflood inundation processes by numerical calculations of our new flood simulation code (see [4] for a detail of our code).

Theoretical view of the Manning coefficient: First, we follow the traditional discussion of turbulence [e.g., 5, 6] to derive a theoretical expression of the Manning equation.

Considering a time-averaged velocity u_a and a velocity fluctuation u^+, the velocity u at a certain depth can be written as:

$$u = u_a + u^+. \tag{1}$$

Using the mixing length l like the mean free path in the kinetic theory of gases, Prandtl hypothesized that the average of the absolute value of velocity fluctuations is proportional to the velocity gradient:

$$\overline{u^+} \sim \overline{w^+} \sim l\frac{\partial u_a}{\partial z}, \tag{2}$$

where w^+ is the vertical velocity fluctuation. The vertically carried momentum in a unit time and unit area is $\rho(u_a + u^+)(w^+)$. This momentum can be considered to correspond to the turbulent shear stress, τ_t, and therefore,

$$\tau_t = -\rho(u_a w^+ + u^+ w^+). \tag{3}$$

Time-averaged values of the fluctuations would equal zero, so that the above equation can be rewritten as

$$\tau_t = -\rho u^+ w^+ = \rho l^2 \left(\frac{\partial u_a}{\partial z}\right)^2. \tag{4}$$

Considering the large Reynolds number for flood flows, the turbulent shear stress much greater than the viscous shear stress. Therefore, the shear stress τ_{zx} can be written as

$$\tau_{zx} \sim \rho l^2 \left(\frac{\partial u}{\partial z}\right)^2. \tag{5}$$

Mixing length l can be considered as a scale of the movement due to the turbulence. Therefore, l should be zero at the bottom of the flow and will be larger in accordance with length from the bottom. This may be expressed as $l = \kappa z$, where κ is called the Karman constant.

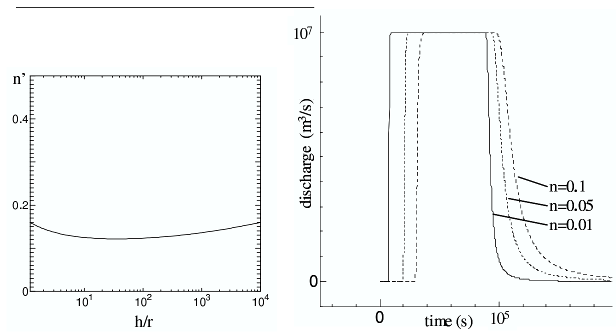

Figure 1: (left) n' versus a wide range of h/r. (right) Calculated hydrographs against time with different Manning coefficients (n=0.01, 0.05, and 0.1).

We assume the shear stress (or the Reynolds stress) is approximately constant throughout the flow and that it has a value equal to the boundary shear stress at the base. These assumptions let us calculate the velocity profile of a turbulent flow. Since the shear stress at the bottom τ_0 can be related to the frictional velocity U_f as $U_f = \sqrt{\frac{\tau_0}{\rho}}$, we can write the velocity as:

$$\frac{du}{dz} = \frac{U_f}{\kappa z}. \tag{6}$$

Integrating this equation, using a scale length of roughness r, we can write

$$u = U_f(\frac{1}{\kappa}\ln\frac{z}{r} + A). \tag{7}$$

Thus, average velocity $\bar{u}$ $\left(= \frac{1}{h}\int_0^h u dz\right)$ becomes

$$\bar{u} = \left(A - \frac{1}{\kappa} + \frac{1}{\kappa}\ln\frac{h}{r}\right)\sqrt{ghS_0}. \tag{8}$$

One of the most famous approaches to predict the average velocity in fluvial hydrology is Chézy's equation, which relates the average velocity to slope as: $\bar{u} = C'\sqrt{hS_0}$. A similar approach, based on experiments, uses the Manning roughness coefficient n [$\mathrm{m}^{-1/3}\mathrm{s}$]:

$$\bar{u} = \frac{h^{1/6}}{n}\sqrt{hS_0}. \tag{9}$$

This equation is often shown to predict water movement very well, and its value is summarized in databases for many natural rivers [e.g., 5]. Comparing this equation and equation (8), we can obtain a theoretical expression of Manning's n as:

$$n = \left(A - \frac{1}{\kappa} + \frac{1}{\kappa}\ln\frac{h}{r}\right)^{-1}\left(\frac{h}{r}\right)^{\frac{1}{6}}\frac{r^{\frac{1}{6}}}{\sqrt{g}} \tag{10}$$

If we write $\left(A - \frac{1}{\kappa} + \frac{1}{\kappa}\ln\frac{h}{r}\right)^{-1}\left(\frac{h}{r}\right)^{\frac{1}{6}}$ as n', we can rewrite this equation as

$$n = n' r^{1/6} g^{-1/2}. \quad (11)$$

Figure 1-left shows n' is nearly constant over a wide range values of (h/r), when we use $A = 8.5$ and $\kappa = 0.4$. For the one order change in depth h, from h/r=100 to 1000, for example, n' varies only 10%. However, if the gravity changes from 9.8m^3/s to 3.7m^3/s, n increases about 60%. Therefore, the above equation suggests that the Manning n is much more strongly dependent on gravity than on the depth of flow. Similar discussion based on a dimensionless drag coefficient can be seen in [7].

Effect of n on a simple slope: Using our semi-3d flood simulation code [4], we calculated flows in a channel under various Manning coefficients to understand its relevance to progressive inundation by a megaflood. We assumed a gentle planar slope, with a size of 100km x 100km, confined on both sides by walls. Water is discharged from a 100km-wide line source, which is located at 30km downslope from the top of the calculation area. Figure 1-right shows calculated hydrographs at the 50km downslope point from the source line for different Manning coefficients. Calculation conditions other than the Manning coefficient are set constant (discharge duration is 1day; discharge rate is 10^{7}m^3/s; discharged water volume is 8.64x10^{4}km^3; and the slope is 0.01). Although the total volume of discharged water is the same, discharge increases rapidly for smaller Manning coefficients because the Manning coefficient represents the flow resistance in the flood. Note that hydrograph recession curves are much steeper for smaller Manning coefficients.

Effect of n on a real topography: The role of the Manning coefficient on a real topography is not as simple as that on a smooth plain or a channel. To illustrate this, we show simple calculations of the Missoula floods using a current-day DEM. We calculated flood flows over the larger area impacted by Missoula flooding. To make the simplest evaluation of this factor, we defined a constant Manning coefficient over the whole area. Figure 2 illustrates some typical results of our calculations, showing the complex influence of the Manning roughness coefficient on flow morphology. Smaller Manning coefficients (Fig. 2-a) make the water flow more efficient, so that the water cannot become deep enough to build high hydraulic potential. Therefore, flooding in this case enters the region without effective widening (Fig. 2-a). However, as seen in Fig. 2-b and c, the flow paths are different when we use larger Manning coefficients. This is because flow that is retarded by higher friction can obtain sufficient hydraulic potential to flow over a topographic barrier.

Discussions: The Manning roughness coefficient is a parameter, that determines not only changes in flow velocity and the shapes of the drainage hydrographs, but also to the depths which contribute to determining particular paths. There is a real question as to the applicability of the empirical Manning coefficient, developed empirically for modern river systems, to a cataclysmic flood which is several orders of magnitude larger than any observed flood. However, as we already discussed, the dependence of Manning coefficient on the depth is quite weak, so that the error included in upscalling the coefficient is less problematic than otherwise might seen apparent. Of course, this presumption is untested because cataclysmic megafloods have never been measured. It is also important to note that the bottom roughness will be changed during a flood event. Erosion and sedimentation depend largely on flow velocity, which is a function of the flow depth. Therefore the bottom condition might be significantly changed in accordance with the variation of the flow depth. This effect is another issue to be considered. For a careful application of the Manning equation, varying the value of the coefficient and checking its performance on a study area are clearly quite important.

References [1] Bretz, J. H. (1925) *J. Geol., 33*, 97-115. [2] O'Connor, J. E. and V. R. Baker, (1992) *Geol. Soc. Am. Bull., 104*, 267-279. [3] Baker V. R., et al. (1993) *Science, 259*, 348-350. [4] Miyamoto, H. et al. (2003) *LPS XXXIV*, this volume. [5] Dingman, S. L. (1984) *Fluvial Hydrology,* pp. 383, [6] Julien, P. (1994) *Erosion and sedimentation*, Cambridge university press, 280pp. [7] Komar, P. D. (1979) *Icarus*, 37, 156-181.

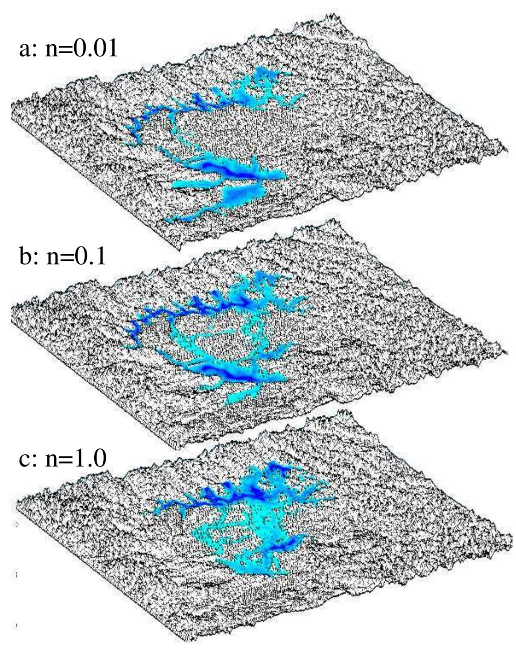

Figure 2: Comparison of areal coverage under various Manning coefficients. a: 2 days later, n=0.01; b: 3.5 days later; $n = 0.1$; and C: 5 days later, n=1.0).