A TEST OF MAXWELL'S Z MODEL USING INVERSE MODELING.

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Introduction: In modeling impact craters a small region of energy and momentum deposition, commonly called a "point source" [1,2], is often assumed. This assumption implies that an impact is the same as an explosion at some depth below the surface. Maxwell's Z Model [3,4], an empirical point-source model derived from explosion cratering, has previously been compared with numerical impact craters with vertical incidence angles, leading to two main inferences. First, the flow-field center of the Z Model must be placed below the target surface in order to replicate numerical impact craters [5]. Second, for vertical impacts, the flow-field center cannot be stationary if the value of Z is held constant; rather, the flow-field center migrates downward as the crater grows [6].

The work presented here evaluates the utility of the Z Model for reproducing both vertical and oblique experimental impact data obtained at the NASA Ames Vertical Gun Range (AVGR). Specifically, ejection angle data obtained through Three-Dimensional Particle Image Velocimetry (3D PIV) [7,8] are used to constrain the parameters of Maxwell's Z Model, including the value of Z and the depth and position of the flow-field center via inverse modeling.

Maxwell's Z Model: Maxwell's Z Model is an empirical model based on explosion cratering data and assumes that the flow-field center is located at the target surface. The ejection angle is given by $\theta_e=\tan^{-1}(Z-2)$ for vertical impacts. If Z is assumed to be constant throughout the majority of crater growth, then ejection angles also should be constant. Usually, ejection angles are taken to be the same as the angle made by the expanding ejecta curtain and the target surface, generally assumed to be 45˚ which leads to a Z value of 3. Previous studies [5,6] compared the Z Model to numerical impacts and determined that the flow-field center must be located approximately one projectile radius below the target surface and that the flow-field center migrated in the vertical direction as the crater grew (for 90˚ impacts). Croft [9] generalized the Z Model to include a term for the depth to the flow-field center (Figure 1) yielding the following formula for ejection angle:

$$\theta_e = \tan^{-1}\left[\frac{\dfrac{\tan\Delta}{\cos\Delta} - \tan^2\Delta + (Z-2)}{\dfrac{1}{\cos\Delta} - \tan\Delta(Z-1)}\right]$$

where Δ depends on the ejection position at the surface (x_e, y_e), the ejection time (t_e) and the depth to the flow field center (d).

In this way, a flow field generated by the Z Model and located at some depth beneath the target surface can predict ejection angles at any location during the excavation stage of an impact.

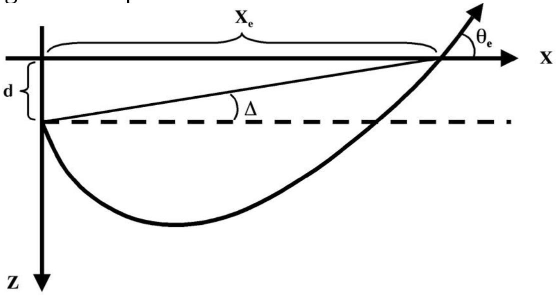

Figure 1. Sketch of variables used in the modified Z Model that incorporates the depth of the flow-field center. (Figure modified from [9].)

Three-Dimensional Particle Image Velocimetry: 3D PIV measures the three-dimensional velocity and position of ejecta particles within a growing ejecta curtain. The system projects a laser sheet parallel to and above the target surface during experimental impacts at the AVGR. Two cameras image the illuminated ring of particles within the ejecta curtain twice in rapid succession such that the particles within the laser plane in the first image have moved only slightly. Processing software tracks the motion of ejecta particles between time frames and combines the two camera views to obtain three-dimensional velocities of particles within the laser plane. The measured velocities are extrapolated back to the target surface to determine ejection positions, velocities and angles for ejecta particles in all directions around the impact point, ideal for oblique impacts.

Ejection positions and times measured using 3D PIV are incorporated into various inverse models in order to constrain the parameters of the modified Z Model. The ejection angles predicted by these models are directly compared to the ejection angles observed using 3D PIV. This strategy tests the applicability of the point-source approximation for both vertical and oblique experimental impacts. In all cases, the problem is assumed to be symmetric about the plane defined by the projectile trajectory and normal to the target surface (referred to below as the trajectory plane).

Constant Z Model: One flow field is present beneath the target surface and allowed to migrate along a subsurface trajectory (figure 2). Z is required to be constant in time and azimuth about the flow-field center. The model parameters derived in this case are the initial depth of the flow-field center as well as its initial location within the trajectory plane, the constant rate and angle at which the flow-field center is allowed to migrate beneath the target surface, and the constant value of Z.

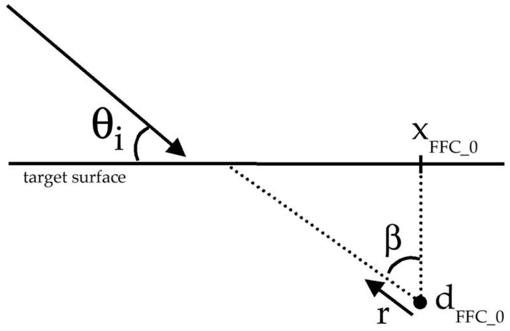

Figure 2. Sketch of model parameters for one Z Model flow field. The projectile trajectory is shown by the arrow and the incidence angle is θ_i. The flow-field center has an initial position (x_{FFC_0}) along a line contained within the trajectory plane and an initial depth (d_{FFC_0}). The flow-field center is allowed to migrate at a constant rate (r) and angle (β) along a subsurface trajectory also within the trajectory plane.

Varying Z Model: The geometry of this case is the same as the constant Z Model, but allows the value of Z to vary linearly in time. This solution was suggested through comparison of Z Model results with numerical models [10] where the Z value was initially 2.0 and increased through half of crater growth to 3.0 for vertical impacts. Previous 3D PIV data [11] as well as an earlier study by Cintala et al. [12] show that ejection angles vary with ejection time and position for both vertical and oblique impacts, supporting either a moving flow-field center at depth or a changing Z value with time.

Azimuthal Z Model: The geometry of this case is the same as the constant Z Model, but allows the value of Z to vary in azimuth around the flow-field center. Previous data collected via 3D PIV [13,14] show an azimuthal dependence of ejection angle during oblique impacts which implies that the Z value may also be dependant on azimuth.

Two Superimposed Z Models: In this case, two constant Z flow fields are allowed to evolve beneath the surface during the impact. Each flow field predicts an ejection angle at any given position. These two ejection vectors are then added to determine the predicted ejection angles at the positions of the observed data. A combination of two superimposed flow fields may reflect the material motions that arise during the transfer of energy and momentum from projectile to target during oblique impacts. This transfer is exposed by ejecta asymmetries during oblique impacts, but is generally masked during vertical impacts.

Implications: The purpose of this study is to evaluate whether the Z Model, modified in any of the four discussed methods, can be used to accurately describe the excavation of vertical and oblique impacts as measured experimentally. In all four models, the location of the flow-field center is allowed to move beneath the target surface. This migration could account for the variation in ejection angles with crater growth in both vertical and oblique impacts. Variation of the value of Z, with either time or azimuth around the impact point, also may account for these observations. Finally, a superposition of two Z Models may provide the most realistic description of the cratering flow field and have implications regarding the transfer of energy and momentum during the initial stages of impact. This application of the Z Model provides a new three-dimensional description of the cratering process for oblique and vertical impacts.

The Z Model approximation could greatly simplify numerical models of crater excavation, especially during the majority of crater growth – i.e., after the shock wave has passed through the target and material motions have been established. Further, it may be possible to apply the Z Model to calculations for oblique impacts, relieving some of the hardware/software constraints for a fully three-dimensional assessment of the excavation flow. Such an approach will not replace well-designed forward numerical models but can provide simplified approximations for specific applications, for example by incorporating scalable descriptions of more unusual targets such as very low-density materials.

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