COULD FAST ROTATOR ASTEROIDS BE RUBBLE PILES?

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Introduction: The strength of asteroids is an important property for dynamical processes involving evolution, disruption, cratering and mitigation methods. Internal stresses are produced by the centrifugal and gravitational forces, which depend on the asteroid's shape and size, and are limited by the strength. Therefore, studies of spin states can be used to infer limits on strength. An oft-quoted result of [1] and others is that a rubble pile, strengthless asteroid cannot spin faster than 11 rev/day, or have a period less than about 2.1 hours. Faster spins produce centrifugal tensile forces at the equator greater than compressive gravity forces, so material would be flung off. That limit is too high and can be refined.

A stress analysis of rotating asteroids: A much more general analysis of the permissible spin is given in [2]. First, a more precise definition of the meaning of "strength and strengthless" is involved. In a particulate geological material such as a soil, "strength" comes from two factors. One is the cohesion: the strength at zero confining pressure; it depends on cohesive forces between particles. Those arise from many different mechanisms. Separate is the "strength" induced by pressure, often called the "friction" term. A soil with confining pressure has a high shear strength because particles are pushed together and interlock, so are not free to slide over one another.

In soil mechanics these two effects are a part of the Mohr-Coulomb strength model, and the friction term defines the material property called the friction angle. In [2] I give a solution for the complete 3D stress states of a self-gravitating, granular, ellipsoidal asteroid assuming that the cohesion, but not the friction angle, is zero. That stress state can be used to determine the maximum spin within the strength capability. Figure 1 from [2] shows the maximum equilibrium spin states for a Mohr-Coulomb model, and data for all large asteroids. The allowable spin rates are a function of both the asteroid aspect ratio and the friction angle of the material. All data shown is within limits for reasonable friction angles. These maximum spin rates are independent of asteroid size, and, for any friction angle, are always less than the simpler tensile strength analysis result of 11 rev/day shown at the top right.

It is the shear stress, not the tensile stress that limits the spin. A spherical asteroid of density 3 would exceed its tensile limit if its period was less than 2.1 hours, but would exceed its shear limit if the period were less than 2.6 hours. One with an elongation of 4:1 is limited to a period no less than 5 hours. The maximum spin is reduced further for smaller mass density. Furthermore, if an asteroid were to be spun up by some mechanism (an impact?) to greater than its equilibrium spin, global shearing on planes 45° to the long axis would elongate the shape and again equilibrium would be achieved. The fig. 2 shows curves of constant angular momentum superposed on the limit curve for the friction angle of 30°, and with elongation the spin would reduce decrease along to the paths shown. That process would always occur before reaching a spin state where material was actually flung off. Spin disruption would not seem to be a possible mechanism for rubble pile asteroids

Small fast rotators: Now, what about smaller asteroids? Figure 3 from [3] shows a plot of many asteroid spin states versus asteroid diameter. All asteroids larger than 1km (and all except one larger than 150 m) are below the critical spin limits noted above. However, all recently discovered asteroids less than 150 m in diameter have spins significantly in excess of that speed limit, as does one at an estimated 0.9 km [4]. They are often referred to as ‘monolithic rock fragments’.

Interpretations in [5] of the fast rotators refer to a sharp cutoff in spin rates, a dramatic separation from the larger bodies, and a fundamental change of physics in this size range. Clearly those asteroids must have cohesive strength, but how much? I report here an analysis extending that of [2] to include both cohesion and friction components of strength; which will be presented at the conference. In this more general case the results for maximum spin depend on friction angle and cohesion, and on both shape and size, so can be plotted in several ways. Assuming a spherical shape, the limits depend only on friction angle, cohesion and size. These limits as a function of diameter for the typical friction angle of 30° and for various cohesion values are shown superposed on the data in figure 3.

These results are very interesting. First, just as for impact cratering, there are two regimes. For the larger asteroids, the cohesion is of no consequence for the range of cohesion considered here. The ‘strength’ of these asteroids is a result of the angle of friction and the gravitational compressive pressures. Further, there is no size dependence in this “gravity regime”. But, for the smaller asteroids in which the gravitational pressures are small, the results depend only on the cohesion, in what can be called a “strength regime”. Here the allowable spin scales as the inverse of the size.

Even more important, note the cohesion required for fast rotators. A cohesion of only 10^4 dynes/cm^2 is all that is required for the observed spins. While an elongated body requires a little more strength, a value of, say, a few times 10^4 dynes/cm^2 is sufficient to hold all of these bodies together! That is three orders of magnitude less that the in-situ value of a terrestrial soil such as desert alluvium. A sample of a material with a cohesion of only 10^4 would collapse under terrestrial gravity if it were over about 5 cm in height!

This result shows that calling these bodies "monolithic" and "shards" and "rocks", while it might be true, is not warranted from the actual data. Statements in the literature referring to the asteroids smaller than a few hundred meters in diameter as "intact, internally monolithic bodies that retain the tensile strength to rotate at such extreme rates" may be misleading. Instead, these small asteroids might also be essentially rubble pile bodies (depending on the particulars of ones favorite definition) with only an almost negligible strength. A Mohr-Coloumb model with a cohesion of a few times 10^4 dynes/cm^2 gives a bound on spin that is clearly above all observations, and smoothly connects the data from the small to the large asteroids. These curves dispel the interpretation of a sharp cutoff, the curves smoothly decrease from the data for the fast rotators right down to the large ones. The recently discovered 2001OE84 [4] is on the same curve as the faster of the smaller rotators. Perhaps all asteroids have some minimum cohesion, although it is unclear what mechanisms might lead to that cohesion for a previously fractured reaccumulated asteroid. It could be as simple as some interlocking mechanism of irregular sized and shaped pieces. In any case, the transitions observed are entirely consistent with a smooth transition from strength scaling to gravity scaling.

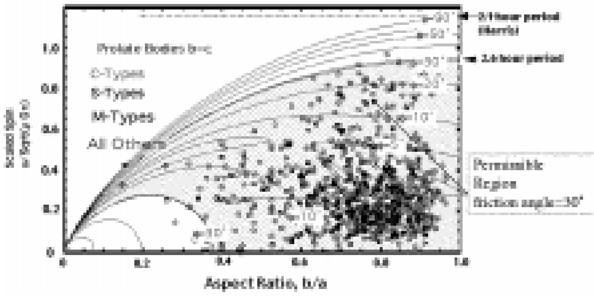

Figure 1. Limit spin states for a cohesionless Mohr-Coloumb ellipsoidal body, for different angles of friction. The highlighted region shows the range of permissible states for an angle of friction of 30˚. The maximum spin has a period of 2.6 hours, not the 2.1 hours for tensile mass shedding, and the maximum period increases further with increasing elongation. Superposed are the data for about 800 asteroids segregated into four taxonomic groups. These are the larger asteroids, the recently discovered small fast-rotators are off this curve at the top. It is seen that all of these large asteroids are within the spin limits determined, consistent with the rubble pile model.

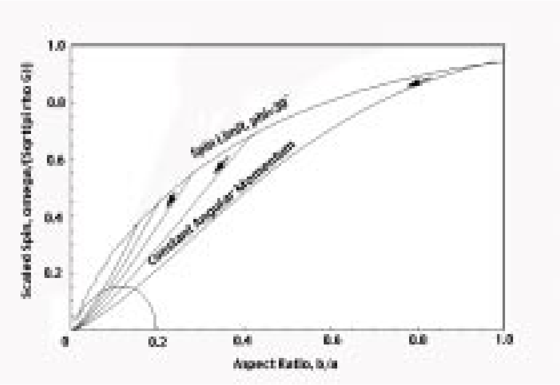

Figure 2. Curves of constant angular momentum and the limit spin curve for 30˚ friction angle. A state with spin above the limit curve will result in global shearing to a more elongated shape. Increased elongation increases the angular moment of inertia, so the spin decreases, to change the state of an asteroid from a spin above the equilibrium limit to one beneath it. This shearing deformation equilibration always occurs at a significantly lower spin than that for mass shedding at the equator.

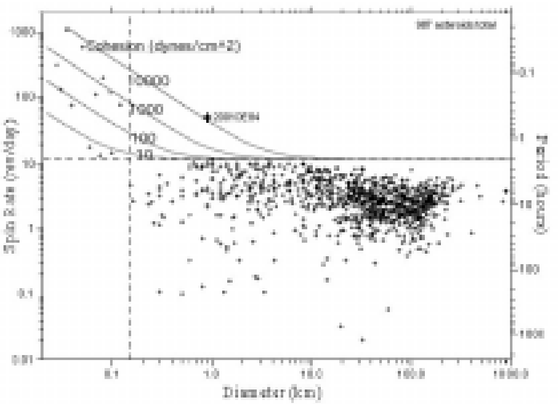

Figure 3. Data and spin limit curves for spherical asteroids as a function of size for a cohesive, Mohr-Coloumb material with different values for the cohesion and an angle of friction of 30˚. The Pravec data for actual asteroids is superposed. The fast rotators are the dozen points at the upper left. A cohesion of 10^4 dynes/cm^2 is sufficient for the equilibrium of all asteroids including the "fast rotators".

References: [1] Harris, A. W. (1996) The rotation rates of very small asteroids: Evidence for "rubble-pile" structure. *Proc. Lunar Planet. Conf. 27th,* 493-494 [2] Holsapple, K.A. (2002) "Equilibrium Configurations of Solid Ellipsoidal Cohesionless Bodies", *Icarus*, Volume **154**, Issue 2, pp. 432-448
[3] Pravec, P., Harris, A. W., and T. Michalowski, (2003) "Asteroid Rotations" *in Asteroids III* [4] Pravec P. and Kusnirak P. (2001) *"2001OE$_{84}$",* IAU Circular 7735.
[5] Whitely R. J., Tholen JD.J. and Hergenrother C. W. (2002) "Lightcurve analysis of four new monolithic fast-rotating asteroids" *Icarus* **157**, 139-154.