DISCONTINUITIES IN SIZE-STRENGTH SCALING LAWS: ANOTHER SOURCE OF WAVY SIZE DISTRIBUTIONS.

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Introduction: Previous studies with numerical collisional models have demonstrated that wave-like departures from traditional, linear power-law equilibrium size distributions can result from a small-size cutoff in the particle population [1–4]. At small sizes, a particle cutoff may result from the rapid removal of bodies by non-gravitational forces such as Poynting-Robertson drag or the Yarkovsky effect.

Another mechanism inducing waves in an equilibrium size distribution is the distinct change in slope in a size-strength scaling law resulting from the transition from strength-scaling for bodies smaller than about 100–200 m in diameter to gravity-scaling for larger objects [5]. As the strengths of larger bodies increase relative to what they would have been had the strength-scaling law continued to larger sizes, their collision lifetimes increase and these “excess” objects represent an overabundant source of projectiles for larger objects. Thus, a wave propagates up the size distribution starting at the size corresponding to the minimum in the size-strength scaling law.

In this study, I demonstrate that discontinuities in the size-strength scaling law, as might occur at small sizes when particles are no longer homogeneous over the scale of fragmentation, can also induce significant waves in evolved size distributions.

Discontinuous Scaling Laws: Figure 1 shows four hypothetical scaling laws which define the critical specific energy, Q^*_D, for bodies from 100 μm to 200 km in diameter. Q^*_D is the energy per unit target mass required to fragment and disperse the target, leaving a largest remnant with 50% the mass of the original target. Scaling law H1 defines the ‘reference’ scaling law for this study, having a strength-scaling-like $D^{-0.4}$ size dependence for bodies smaller than $D \approx 100$ m and a gravity-scaling-like $D^{1.1}$ size dependence for bodies larger than $D \approx 100$ m. At size $D = 10$ cm $Q^*_D = 8100$ J/kg, equivalent to the intermediate strength material of [6]. Scaling law H1 is similar to scaling law H2 in Fig. 3 of [5], with the critical specific energy a factor of ~1.5 smaller at all sizes. Here, scaling law H1 assumes that the same $D^{-0.4}$ size dependence of Q^*_D extends to the very smallest, dust-size particles. Most other studies have focused on the collisional evolution of comparatively larger objects and have not addressed the issue of the strength properties of very small particles.

Scaling laws H2, H3, and H4 of this study are all identical to scaling law H1 for sizes larger than $D = 1$ cm. At smaller sizes, however, I allow for the possibility that the critical specific energy may change abruptly due to changing material properties at small size scales (see Discussion below). The actual size at which such a change in material properties occurs may be quite different than the 1-cm scale assumed for the convenience of this numerical study.

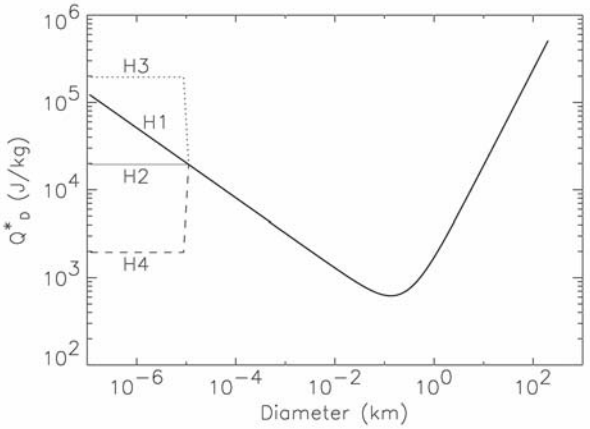

Figure 1. Four hypothetical scaling laws used to explore the behavior of a collisional model with abrupt and/or discontinuous changes in critical specific energy.

In scaling law H2, the Q^*_D variation with size is assumed to change abruptly and remain constant for $D <$ 1 cm at the same ~1.95×10^4 J/kg value as for 1 cm diameter particles. In scaling law H3, Q^*_D is assumed to increase discontinuously by a factor of 10 and remain constant. In scaling law H4, Q^*_D is assumed to decrease discontinuously by a factor of 10 and remain constant.

Collisional Model Results: The four hypothetical scaling laws described above were used within the same collision model described in [1,4] to determine catastrophic disruption lifetimes and resulting evolved size distributions. Initial populations with a $D = 200$ km largest body and a power-law slope index of $p = 2.7$ (the number of bodies larger than D is proportional to D^{-p}; for scaling laws with size-independent Q^*_D, equilibrium size distributions have $p = 2.5$) were followed through 4.5 Gyr of collisional evolution.

Figures 2–4 show the resulting evolved size distributions. In each case, the evolved size distribution obtained with reference scaling law H1 is plotted with a heavy solid line for comparison.

Figure 2 shows that an abrupt change in the slope of the log Q^*_D vs. log D relation (in this case from a strength-scaling-like $D^{-0.4}$ size dependence to an energy-scaling, size-independent strength at smaller sizes) induces minor perturbations to the evolved size distribution. The equilibrium power-law slope index changes value at the size where the scaling law changes slope, and relatively minor wave-like perturbations propagate up the size distrubtion from that point.

Figures 3 and 4 show that a discontinuity in the size-strength scaling law can result in significant wave-like perturbations in evolved size distributions. The wavelength of the waves is determined by the critical projectile-to-target size ratio set by the size-strength scaling law. The amplitude of the waves depends on the magnitude of the discontinuity in the log Q^*_D vs. log D relation. A larger magnitude discontinuity leads to larger amplitude waves. The phase of the waves depends on the nature of the discontinuity. An abrupt change to *stronger* particles at smaller sizes leads to an immediate *trough* in the size distribution for particles just larger than that at which the strength properties change, since the stronger particles survive longer and represent an overabundant source of projectiles to break up larger bodies. An abrupt change to *weaker* particles at smaller sizes leads to an immediate *crest* in the size distribution, since the weaker particles do not survive as long and represent an underabundant source of projectiles.

Discussion: Much attention has been focused on determining how Q^*_D varies with size for objects larger than the ~10-cm size targets typically studied in laboratory impact experiments. This is understandable as impact studies and collisional models have traditionally focused on the evolution of populations of larger objects where telescopic data can constrain the models. However, now that we are beginning to understand the extent to which the details of small-particle removal mechanisms and the effects of changes in size-strength scaling relations can affect the evolved size distribution at even the largest sizes, the time has come for a better understanding of the impact properties of particles smaller than ~10 centimeters.

At size scales small enough that material inhomogeneities due to the presence of chondrules or Ni-Fe metal blebs or the effects of mineral grain boundaries become important, impact strengths may be dictated by the failure of internal flaws with a narrow range of critical tensile strengths [7]. Q^*_D may still depend on size, but may change discontinuously at specific sizes and/or have a different size dependence in different size ranges.

References: [1] Durda D. D. (1993) Ph.D. thesis, Univ. of Florida. [2] Davis D. R. et al. (1993) *Lunar Planet. Sci.,* 24, 377–378. [3] Campo Bagatin A. et al. (1994) *Planet. Space Sci.*, 42, 1079–1092. [4] Durda D. D. and Dermott S. F. (1997) *Icarus*, 130, 140–164. [5] Durda D. D. et al. (1998) *Icarus*, 135, 431–440. [6] Davis D. R. et al. (1985) *Icarus*, 62, 30–53. [7] Durda D. D. and Dermott S. F. (1996) *ASP Conference Series Vol. 104*, 473–476.

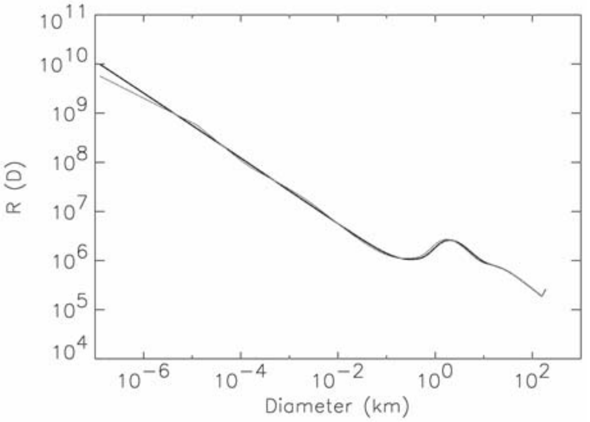

Figure 2. Evolved size distributions for reference scaling law H1 (heavy solid line) and scaling law H2 (light solid line).

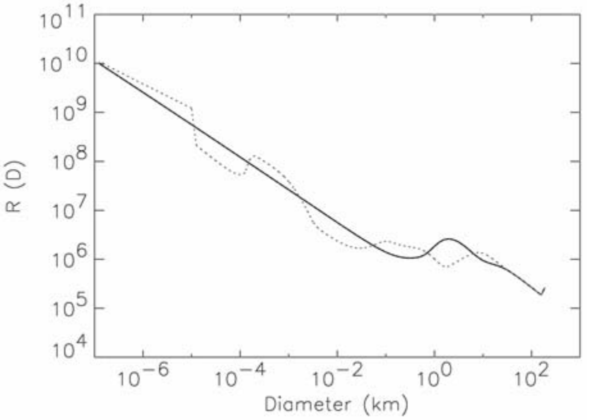

Figure 3. Same as Fig. 2, but for scaling law H3 (dotted line).

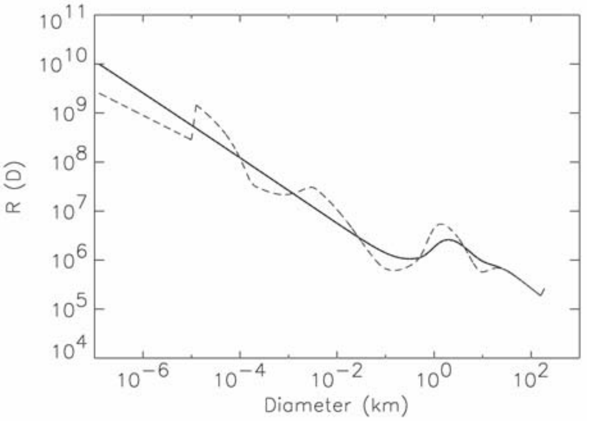

Figure 4. Same as Fig. 2, but for scaling law H4 (dashed line).