Partial Resetting on Hf-W System by Giant Impacts. T. Sasaki\textsuperscript{1} and Y. Abe\textsuperscript{1}, \textsuperscript{1}Department of Earth & Planetary Science, Graduate School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan, take\textsuperscript{1}or@eps.s.u-tokyo.ac.jp.

1. Introduction: Hafnium and tungsten are both highly refractory elements, and hafnium is a lithophile element whereas tungsten is a moderately siderophile element that should strongly partitioned into metal phases during metal/silicate segregation. Hf-W chronometry provides constraints on the timing of planetary accretion and differentiation, as the segregation of a metal core from silicates should induce strong fractionation of hafnium from tungsten. In previous studies, it was assumed a giant impact raise up perfect resetting on Hf-W chronometer. Recently the effect of partial resetting on Hf-W system is starting to be considered [1, 2].

2. Mechanisms of core formation by giant impacts: In the stage of giant impacts, newly accreted W of the impactor’s metal equilibrates with W in the silicate melt of the target during the sedimentation of metal in silicate mantle. Suppose that metal of the impactor split into lots of metal sphere with a power law size distribution sinking at the Stokes velocity. When the cumulative power law exponent b = 1, the radius of metal sphere should be less than about 50 centimeter to achieve the perfect resetting of the chronometer.

Recently many works have modeled giant impacts using a method known as smooth particle hydrodynamics, or SPH [3]. However, these calculations don’t have enough resolutions to discuss the size smaller than meter-order. The estimated size of metal droplets sinking in silicate melt is about 1 centimeter owing to splitting of metal by Rayleigh-Taylor instability [4]. This is small enough to achieve perfect resetting.

However, even after the splitting of metal layer to small droplets, there is the possibility of Rayleigh-Taylor instability between the layer with metal droplets and that of metal free [5]. Such type of Rayleigh-Taylor instability is observed in laboratory experiments [6]. The schematic view of such Rayleigh-Taylor instability is shown in Fig.1. Fig.1 shows the regions where metal droplets are not interact with silicate. Such a region appears because the growth of the Rayleigh-Taylor instability is by far faster than the Stokes sedimentation of metal droplets. Thus, complete metal-silicate equilibration cannot be expected.

3. Model calculations: The simplest model of the fractionation is a two-stage model with a single episode of core formation at the age t = \( \Theta \) occurring sometime after the origin of the solar system. Then the isotopic time evolution of \( \Theta \) is given below [7].

\[
\frac{d\Theta}{dt} = Q \cdot f \cdot \frac{^{182}\text{Hf}}{^{180}\text{Hf}} \cdot \left[ e^{\frac{t}{\Theta}} \right] \cdot e^{\frac{t}{\Theta}}
\]

In this equation, \( \Theta \) is the \(^{182}\text{Hf} \) decay constant \( \textnormal{(0.077 Myr}^{-1}) \), and \( Q = 1.55 \times 10^7 \). The Hf/W fractionation in a reservoir relative to Chondritic Uniform Reservoir (CHUR) is defined by \textit{f-value}, \( f = 12 \) for the region of Earth formation, and \( (^{182}\text{Hf}/^{180}\text{Hf})_\text{Earth} = 1 \times 10^{-4} \) [8].

To calculate the isotopic evolution of Hf-W system, we consider two stages for planetary accretion: (1) the protoplanets formation stage and (2) the giant impacts stage.

In the protoplanets formation stage, the isotopic fractionation is given by

\[
\frac{d\Theta}{dt} = k \cdot \frac{dV}{V} \cdot (\Theta(t)) + \int k \cdot \frac{dV}{V} \cdot \Theta
\]

where \( V \) is the mass of the target body, \( dV \) is the mass of each planetesimal. The degree of the equilibration by each planetesimal impact, \( k \), is a parameter in this model. We iterate this calculation for a hundred million times. A plot of \( \Theta \) versus the age of the protoplanet formation is shown in Fig.2. This results in \( \Theta \) at 10 Myr of 10-12.

In the giant impacts stage, each giant impact partially equilibrates the target body and, thus, partially resets its chronometer. The \( \Theta \) of the protoplanet is given by

\[
\Theta_i = p \cdot (\Theta(t)) + \int (1 - p) \cdot \Theta
\]

A value for the partial resetting ratio, \( p \), and the number of giant impacts, \( n \), are parameters in this model. We assume the first giant impact occurred at 10 Myr, so the initial value of \( \Theta \) is fixed at 10. Subsequent giant impacts are assumed to occur at even intervals. In this study, we estimate the resetting ratio \( p \) that is required for fitting the Earth’s observational data (\( \Theta = 2 \)) (Fig.3). Fig.3 shows that we cannot determine the giant impact age or the
metal-silicate separation age with Hf-W chronometry without a quantitative assessment of equilibration ratio and the number of giant impacts. On the other hand, this result indicates that the resetting ratio of each giant impact is required to be greater than 0.2.

4. Discussions: A giant impact ejects and vaporizes the mantle of the protoearth to a certain depth. The ejecta is mixed and equilibrated with debris of broken impactor before falling back to the protoearth. According to Fig.1, the resetting ratio by a giant impact largely depends on the volume of the protoearth’s mantle that are ejected or vaporized. Since the resetting ratio at each giant impact must be larger than 0.2, we conclude that more than two-tenth of the protoearth’s mantle should have been ejected or vaporized by a giant impact.

The core formation event of Mars can be discussed from the viewpoint of Hf-W chronometry. We calculate the isotopic evolution of $\epsilon$ in the Mars region with $f$-value = 2. This results in $\epsilon = 3-4$ at 10 Myr, which are significantly larger than the $\epsilon$ value for Mars, $\epsilon = 2$. It indicates that Mars must have experienced certain equilibration event after the formation of protoplanets. The required resetting ratio of this event for Mars’ $\epsilon$ is larger than 0.3. Therefore, Mars should have experienced an event, which equilibrate more than three-tenth volume of Mars’ mantle, such as core formation with mantle overturn or a single giant impact.

Fig1: Sketch of metal droplets’ behavior in a mixture of metal droplets and silicate melt. While each droplet is sinking by Stokes sedimentation, the mixture layer is falling into the core as a cluster.

Fig2: Isotopic evolution of $\epsilon$ as a function of the age of protoplanet formation in the Earth’s region. Parameter $k$ is 0.1, 0.5, 1, 5, 10 from top to bottom.

Fig3: Required resetting ratio for fitting the observed value of Earth’s samples as a function of Earth formation age. The number of giant impacts is 1 to 10 from top to bottom. Initial state is $\epsilon = 8$ and $t = 10$. Total time for perfect resetting (resetting ratio = 1) is about 30 Myr, corresponds to previous study (Yin et al. 2002).