A PLAUSIBILITY OF Z-MODEL. K. Wada[1], H. Senshu[2], and T. Matsui[3], [1]*Department of Earth and Planetary Science, University of Tokyo, Tokyo, 113-0033, Japan (wada@eps.s.u-tokyo.ac.jp)*, [2]*IFREE, JAMSTEC, Kanagawa, 237-0061, Japan*, [3]*Graduate School of Frontier Sciences, University of Tokyo, Tokyo, 113-0033, Japan*.

Introduction: The Maxwell's Z-model[1] has been used as a first-order approximation model of excavation flow for vertical impact (e.g., [2]). It can be derived analytically from a few simple assumptions: (1) Flow is steady and (2) incompressible. (3) Origin of flow (hereafter we call it effective origin of flow (EOF)) is a point source. (4) The radial component of particle velocity, u_r, decreases exponentially with increasing the radial distance r, as $u_r \propto r^{-Z}$. Z is a parameter that determines the rate of attenuation of u_r with distance. These assumptions give the steady streamlines in polar coordinates (its origin is given by EOF) as $r = r_0(1-\cos\theta)^{\frac{1}{Z-2}}$, where r_0 is a constant given for each streamline and θ is measured from the vertical downward line.

A plausibility of the Z-model has not been studied so far, because the motion of target materials is not visible directly in laboratory experiments. In numerical simulations of impact cratering [3, 4, 5], the velocity field can be calculated at each time step of cratering. According to these numerical studies, the value of Z is estimated to be about 3, though varying a little bit with time, and the depth of EOF is located at about one projectile diameter below the target surface. These simulations were carried out using a hydrocode with finite difference scheme. Thus, the actual motions of target particles were not able to be traced.

We have conducted numerical simulations of impacts onto granular materials using the Distinct Element Method (DEM) and demonstrated that this simulation method can simulate well the cratering process for the vertical impact [6]. Based on the DEM, the trajectories of target particles can be traced directly, because the motions of distinct particles are computed by this method. It is thus possible to compare the actual motions of target particles with the streamlines predicted by the Z-model.

Distinct Element Method: DEM has been developed to reveal the behavior of granular materials (e.g., [7]). In the DEM, particles are considered as spheres, which are not allowed to deform but overlap a little bit each other. The motion of each particle at each time step is calculated by solving the equation of motion of each particle. The mechanical interaction forces between contact particles is assumed to be expressed by elastic force and friction, modelled by the Voigt-model, which consists of a pair of a spring and a dash-pot. The spring and the dash-pot express elastic force based on the Hertzian elastic contact theory and energy dissipation during contact, respectively. In addition, a friction slider is introduced for the tangential direction to express Coulomb's friction law. Cohesion and rotational resistance are not included in our simulation. These interactions are essentially parameterized by the coefficients of restitution e and friction μ.

Initial condition and parameters: As a granular target, 384,000 equal-sized spherical particles (radius, 1mm; density, 2.7g/cm^3; Young's modulus, 94GPa; Poisson's ratio, 0.17) are randomly placed in a rectangular container (20cm×20cm×7cm). A cross sectional view of the target is shown in Fig. 1. We prepared such a target by dropping the particles and calculating their movements until the motions of the particles settle down. The porosity of the target is about 43%. The coefficients of restitution of walls of the container is set to be 0 so that the reflection waves cannot be generated at the walls.

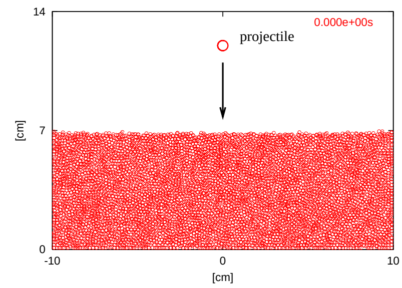

Figure 1: A cross sectional view of the target before impact. Only the particles whose center is located within this cross section (4mm thick) are shown.

A projectile particle (radius, 3mm; density, 2.7g/cm^3; Young's modulus, 70GPa; Poisson's ratio, 0.35) impacts vertically into the target at velocities of $V_i = 100$, 300 or 500m/s, respectively. The acceleration due to gravity is assumed to be 9.8m/s^2 in this simulation. The coefficient of restitution and friction between particles are changed as $e = 0.1, 0.4, 0.8$ and $\mu = 0.1, 0.5, 0.9$.

Numerical results: The numerical results on the excavation process are almost independent of the parameter values varied in the simulation, such as e and μ. Thus, in the following we show the results for the representative case ($V_i = 300$ m/s, $e = 0.4$ and $\mu = 0.5$).

In Fig. 2, we show the initial positions (red circles) of the target particles ejected across the original target surface. This region is considered to be the net excavation region. We also show the Z-model's streamline ($Z = 3$; EOF is located at the target surface (EOF$= 0$)) which crosses the edge of the transient crater. According to the Z-model, the net excavation region is given by the region above this line. The numerical simulation suggests that the net excavation region is roughly similar to that of the Z-model. There exist ejected particles initially located out of the net excavation region predicted by the Z-model. These particles are considered to be the particles pushed up during cavity growth and form a part of the crater rim.

If ejected particles move along the streamlines of the Z-model, all the particles located initially along such streamline should be ejected from some specific ejection point at which the streamline crosses the original target surface. Fig. 3 shows the initial positions of

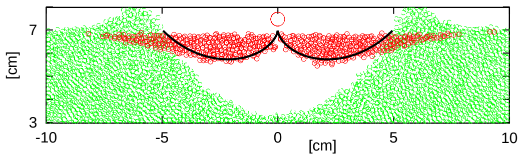

Figure 2: The cross sectional view of transient crater (locations of particles at the real time step are represented by green circles) for the representative case. The initial positions of the particles ejected across the original target surface are represented by red circles. The Z-model's streamlines (for $Z = 3$; depth of EOF$=0$) which cross the edge of the transient crater are shown by black curves. A projectile particle (a large red circle) is also shown for comparison.

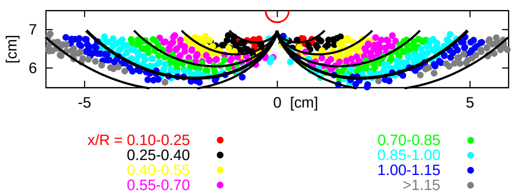

Figure 3: The initial positions of ejected particles with the same ejection point for the representative case . As indicated in the legend, each color corresponds to each ejection point x/R, where x is the horizontal distance from the impact point and R is the transient crater radius. In this case $R = 4.95$ cm. The Z-model's streamlines ($Z = 3$ and depth of EOF$=0$) are represented by black curves. The bold curve represents the streamline that crosses the edge of the transient crater. One half of a projectile particle is shown for comparison at the top of this figure.

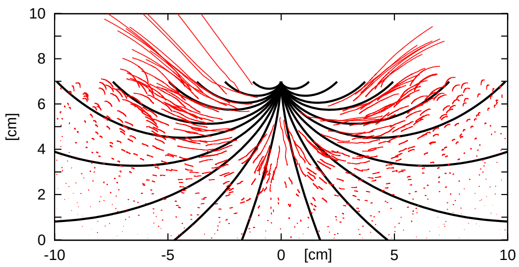

Figure 4: The trajectories of the target particles (red curves) and the Z-model's streamlines (black curves, $Z = 3$ and depth of EOF$=0$) for the representative case. For clear visualization only the trajectories of some arbitrarily-chosen particles are drawn.

ejected particles with the same ejection point. Particles ejected from the same ejection point are represented by the same symbol. As clearly shown in Fig. 3, initial positions of the particles are roughly distributed along the streamlines of the Z-model. This means that the particles located along each streamline are ejected from the identical ejection point. Therefore, it is suggested that ejected particles move along the streamlines predicted by the Z-model.

In Fig. 4 we show the trajectories of target particles. The streamlines of Z-model is also shown as the bold curves. As can be seen in Fig. 4, the particles located at the shallower part of the target have almost similar direction predicted by the streamlines of Z-model. However, such a tendency cannot be observed for the particles located at the deeper region. The reason of this disagreement in the deeper region may be the influence of the target walls.

Discussion: The Z-model streamlines are dependent on Z value and the depth of EOF. We try to fit the Z-model streamlines with various Z value and the depth of EOF. As a result, it is revealed that the excavation flow of our simulation are well expressed by the Z-model streamlines with $Z = 2.7$ and depth of EOF$= a$ (a is the projectile radius), and with $Z = 3$ and depth of EOF$= 0$. Although there exist some differences, the resulting Z value and the depth of EOF are similar to the values obtained in the previous studies [3, 4, 5].

The particles treated in our simulation are non deformable. Therefore one might expect the behavior of the target to be almost incompressible, which satisfies the assumption of the Z-model. It should be, however, noted that the target is granular and porous. Target porosity may make the target compressible. As the target consists of equal-sized spherical particles, the lowest value of the target porosity is about 26%. The porosity in this simulation, about 43%, is higher than this value (this means that the target effective porosity is about 20%). Therefore, our result may suggest that excavation flow, even in a moderately porous medium, can be expressed by the streamlines predicted by the Z-model.

Acknowledgement: Simulations were carried out at the JAMSTEC's MSTSC super computer system.

References: [1]Maxwell, D. E. (1977) in *Impact and Explosion Cratering*, pp. 1003-1008. [2]Croft, S. K. (1980) *Proc. Lunar Planet. Sci. Conf. 11th*, 2347-2378. [3]Thomsen, J. M. et al. (1979) *Proc. Lunar Planet. Sci. Conf. 10th*, 2741-2756. [4]Austin, M. G. et al. (1980) *Proc. Lunar Planet. Sci. Conf. 11th*, 2325-2345. [5]O'Keefe, J. D., and Ahrens, T. J. (1981) *Rev. Geophys. Space Phys.*, 19, 1-12. [6]Wada, K. et al. (2003) *Proc. Lunar Planet. Sci. Conf. 34th*, #1529. [7]Cundall, P. A. and Strack, O. D. L. (1979) *Géotechnique*, 29-1, 47-65.