ON THE FINAL MASS OF GIANT PLANETS. P. R. Estrada, *NASA Ames Research Center, Moffett Field CA 94035, USA, (estrada@cosmic.arc.nasa.gov)*, I. Mosqueira, *NASA Ames Research Center/SETI Institute, Moffett Field CA 94035, USA (mosqueir@cosmic.arc.nasa.gov).*

In the core accretion model of giant planet formation [1], when the core reaches critical mass, hydrostatic equilibrium is no longer possible and gas accretion ensues [2]. If the envelope is radiative, the critical core mass is nearly independent of the boundary conditions and is roughly $M_{crit} \sim 10 M_\oplus$ (with weak dependence on the rate of planetesimal accretion $\dot{M}_{core}$ and the disk opacity κ; [3]). Given that such a core may form at the present location of Jupiter in a time comparable to its Type I migration time ($10^5 - 10^6$ years; [4]) provided that the nebula was significantly enhanced in solids with respect to the MMSN [1] [7] and stall at this location in a weakly turbulent ($\alpha \lesssim 10^{-4}$) disk [8], it may be appropriate to assume that such objects inevitably form and drive the evolution of late-phase T Tauri star disks. Here we investigate the final masses of giant planets in disks with one or more than one such cores. Although the presence of several planets would lead to Type II migration (due to the effective viscosity resulting from the planetary tidal torques), we ignore this complication for now and simply assume that each core has stalled at its location in the disk.

Once a core has achieved critical mass, its gaseous accretion is governed by the Kelvin-Helmholtz timescale [9]

$$\tau_{KH} \simeq 10^b \left(\frac{M_P}{M_\oplus}\right)^{-c} \left(\frac{\kappa}{1\,\mathrm{cm^2 g^{-1}}}\right) \text{ years}, \quad (1)$$

where $b \simeq 10$ and $c \simeq 3$ have been derived [10] by fitting the results of [1]. Then the planetary growth rate due to the contraction of the envelope is given by

$$\frac{dM_{gas}}{dt} \simeq \frac{M_P}{\tau_{KH}}. \quad (2)$$

Although this rate is independent of the boundary conditions, as will be shown, the final mass of the giant planet does depend on the gas surface density Σ of the nebula (unlike the case with the thermal condition criterion of [11]). As the protoplanet grows it opens a gap in the nebula, thereby limiting the amount of gas it can accrete.

Contrary to the standard picture of gap-opening in which the growing protoplanet opens a gap right next to itself, a more realistic treatment, utilizing a damping prescription for tidally induced waves in which the waves launched by the protoplanet shock and damp as a result of their non-linear evolution [8][12], indicates that the gap first forms at a distance away from the protoplanet where the waves first shock. For a constant surface density and vertically isothermal disk, this distance in terms of the local scale height H of the disk is given by [8]

$$l_{sh} \approx \frac{2}{3}\zeta H \left(\frac{M_P}{M_1}\right)^{-2/5} \quad (3)$$

[1] We use the standard MMSN model of [5]. Note that the isolation mass at 5 AU of a disk with $3\times$ MMSN of solids is $\sim 10 M_\oplus$ [6].

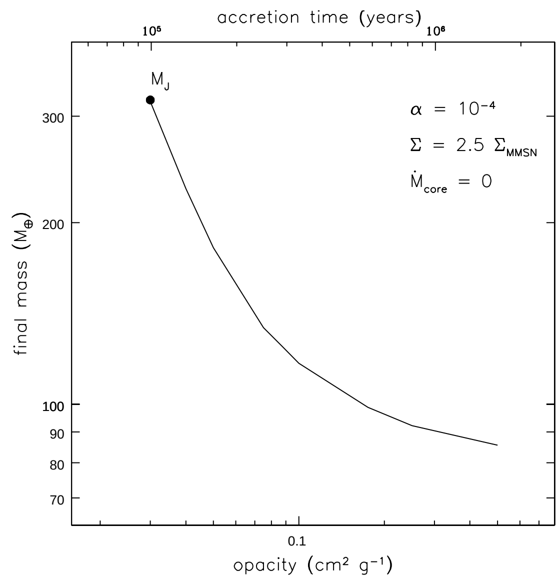

Figure 1: Plot of the final mass of the giant planet versus disk opacity κ for a protoplanet at 5.2 AU. Included are the accretion times.

where $\zeta = 1.4$, $M_1 = (2/3) M_\odot h^3$, and $h = H/r$.

In order to determine the final mass, we have developed a code that incorporates the growth of the giant planet with the disk evolution. The growth of the giant planet is handled using Eq. (2) and by drawing gas from the protoplanet's feeding zone, which we define to be the minimum of the accretion radius $R_a = GM_P/c^2$ (c is the local sound speed) and $2.5 R_H$ (R_H = Hill radius). Combining the continuity and angular momentum equations lead to a self-consistent equation for the evolution of the surface density Σ in terms of the viscous and planetary tidal torque (G and F, respectively)

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{2\pi r}\frac{\partial}{\partial r}\left\{\left[\frac{\partial}{\partial r}(\Omega r^2)\right]^{-1}\frac{\partial}{\partial r}(G - F)\right\}, \quad (4)$$

where Ω is the disk frequency, $G = -2\pi r^3 \nu \Sigma (d\Omega/dr)$, and we solve Eq. (4) using an implicit scheme. For the planetary tidal torque F, we employ the prescription of [12] where the disk is vertically isothermal and the disk response is strictly 2D. F is then expressed in terms of a damping function φ as

$$F(r) = \Gamma (GM_P)^2 \frac{\Sigma_P r_P \Omega_P}{c_P^3}\left[1 - \varphi\left(\frac{r - r_P}{l_{sh}}\right)\right], \quad (5)$$

where $\Gamma = (4/9)\mu_{max}^3(Q)(2K_0(2/3) + K_1(2/3))^2$ [13] and $Q = M_\odot h/\pi \Sigma r^2$ is the Toomre parameter. All quantities with subscript P are evaluated at the location of the protoplanet. The damping function has the functional form

$$\varphi(t) = \begin{cases} 0, & t < t_{sh} \\ \left[1 + (t/t_{sh} - 1)^2\right]^{-1/4}, & t > t_{sh} \end{cases}, \quad (6)$$

where $t = t(\Sigma, r)$, and t_{sh} corresponds to the distance at which the wave first shocks [8][12]. Finally, we lower the total torque by a factor of 5 to account for 3D effects[2].

The gap-opening criterion for an inviscid disk can be expressed as [8]

$$\frac{M_P}{M_1} > \min\left[5.2Q^{-5/7}, 3.8(Q/h)^{-5/13}\right]. \quad (7)$$

In this limit, a Σ of $3.7\times$MMSN and a gap-opening mass of $13M_\oplus$ is obtained at 5.2 AU assuming that a $1\ M_J$ planet derives all of its mass from within the shocking distance l_{sh} of acoustic waves that immediately precedes runaway gas accretion. In this calculation, we have used a passive disk model [15] with temperature profile $T \simeq 165(AU/r)^{3/7}$ K, and a crossover mass of $\sim 30M_\oplus$ [1].

However, steep gradients in the surface density will give rise to a local α due to a Rayleigh-Taylor instability, thus for these preliminary models we chose a nominal $\alpha = 10^{-4}$ in order to avoid sharp gradients as the gap forms. In Figures 1 and 2 we present some initial calculations that demonstrate the dependence of the final mass for an isolated planet on the choice of κ and Σ, respectively, for cases in which $\dot{M}_{core} = 0$. Since the growth rate is a proportional to a larger power of the mass than the tidal torque, the final mass of the planet depends sensitively on the timing of runaway growth. Also, larger κ allows more time for the tidal torque to push gas away from the planet (Figure 1). For a given κ (and α), the final mass of the planet depends on Σ (Figure 2).

We investigate cases with single and multiple cores in the disk. Although a stalled core will begin to open a gap, the protoplanet may still grow to a Jupiter size not only because of the annulus of material within the shocking length of acoustic waves, but also due to the effects of neighboring protoplanets, and the similarity between the gap-opening and growth timescales. We argue that the following parameters may determine the difference between gas-rich and gas-poor giant planets; (a) the disk opacity; (b) the relative timing of runaway between neighboring protoplanets; (c) the value of $\dot{M}_{core}$.

[2] In the prescription of [12], the authors divide the disk into a "launch zone" and a non-linear dissipation zone. They calculate the form of the wake in the linear zone which they allow to damp in the non-linear region (beyond $(4/3)H$ from the protoplanet). The authors ignore the launching of waves in the non-linear portion of the disk. Therefore we supplement this treatment by adding those resonant m-numbers that fall in the non-linear region using the tidal torque formulae of [14].

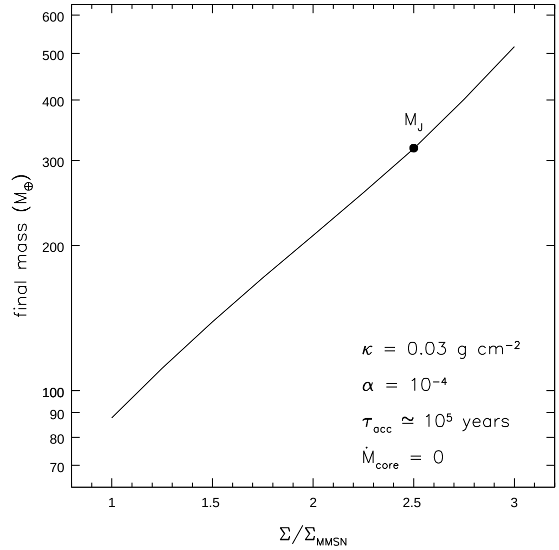

Figure 2: Plot of the final mass of the giant planet versus Σ for a protoplanet at 5.2 AU. For this choice of parameters, we can get a Jupiter mass if Σ at Jupiter's location is $2.5\times$MMSN.

One caveat of these calculations is that 3D effects may increase the value of l_{sh} providing more mass to be accreted by the growing protoplanet, and thus altering the gap-opening criterion. Another is that some angular momentum flux propogates towards the disk surfaces and may act to remove gas within l_{sh} lowering the amount of gas available for accretion.

This work is supported by the National Research Council, and a NASA PGG grant.

References [1] Pollack, J. B., O. Hubickyj, P. Bodenheimer, J. J. Lissauer, M. Podolak, and Y. Greenzweig 1996. *Icarus* **124**, 62. [2] Mizuno, H. 1980. *Prog. Their. Phys. Suppl.* **64**, 54. [3] Stevenson, D. J. 1982. *Plan. & Spac. Sci.* **30**, 755. [4] Bate, M. R., S. H. Lubow, G. I. Ogilvie, and K. A. Miller 2003. *MNRAS* **341**, 213. [5] Hayashi, C. 1981. *Prog. Theor. Phys. Suppl.* **70**, 35. [6] Kokubo, E., and S. Ida 1998. *Icarus* **131**, 171. [7] Inaba, S., G. W. Wetherill, and M. Ikoma 2003. *Icarus* **166**, 46. [8] Rafikov, R. R. 2002. *Astrophys. J.* **572**, 566. [9] Ikoma, M., K. Nakazawa, and H. Emori 2000. *Astrophys. J.* **537**, 1013. [10] Bryden, G., M. Rozyczka, D. N. C. Lin, and P. Bodenheimer 2000. *Astrophys. J.* **540**, 1091. [11] Lin, D. N. C., and J. Papaloizou 1993. In *Protostars and Planets III*, 749. [12] Goodman, J. and R. R. Rafikov 2001. *Astrophys. J.* **552**, 793. [13] Goldreich, P., and S. Tremaine 1980. *Astrophys. J.* **241**, 425. [14] Ward, W. R. 1997. *Icarus* **126**, 261. [15] Chiang, E. I., and P. Goldreich 1997. *Astrophys. J.* **519**, 279.