

**THE EFFECT OF PARAMETRIC RESONANCE ON THE STRUCTURE OF PLANETARY RINGS.** A.E. Rosaev<sup>1</sup>, <sup>1</sup>FGUP NPC "NEDRA" Svobody 8/38, Yaroslavl 150000, Russia, E-mail: hegem@mail.ru.

**Introduction:** Planetary rings are a very complex system. It is a mixture of different-size collided particles. The electromagnetic forces can play an important role. However, in spite of very small mass of particles, gravitational forces between particles always take place and cannot be neglected. The main target of this research is to show, that effect of mutual ring's particles interaction may be significant.

On the other hand, by what way to gravitation may be taken into account? It is natural, to consider motion of the probe particle in the gravitational field of homogeneous ring. Perturbation force in this case calculated as an integral. In the limit, this integral can be substituted by sum with very large  $N$ . At the symmetric circular ring we can apply central configuration model. (The system of  $N$  massive points  $m$ , placed at vertex of regular polygon + central mass  $M$ , is called central configuration). In case of low-collisional ring with large particles, this discrete model has some advantages over continuous model. The model of the central configurations successfully applied to the real planetary rings by Meyer K.R. and Schmidt D.S.[1].

**Parametric resonance:** The equations of planar restricted Hill's problem[2]:

$$\begin{aligned} \frac{d^2 X}{dt^2} - 2m \frac{dY}{dt} + fX &= 0 \\ \frac{d^2 Y}{dt^2} + 2m \frac{dX}{dt} + gY &= 0 \end{aligned} \quad (1)$$

where  $m$  – perturbing body mass,  $X, Y$  – rectangle coordinates,  $f$  and  $g$  – known functions,  $t$  – time. These equations may be reduced to a Hill equation for normal distance from variation orbit  $x$ :

$$\frac{d^2 x}{dt^2} + \omega^2(t)x = f(t) \quad (2)$$

where  $f(t)$  – known function of time,  $\omega(t)$  – periodic function of time, depends from ring perturbation function  $U_r$ .

$$\omega_0^2 = F(R) + \frac{d^2 U_{crg}}{dR^2} \quad (3)$$

where  $F(R)$  – known function of ring central distance  $R$ ,  $U_{crg}$  – central configuration perturbation function.

It means, that area of parametric instability will appear in problem. Then, we enter gravitation interaction between ring particles, by modeling the ring as a system of massive points. We shall consider ring consist of  $N$  attractive particles, placed close to vertex of regular polygon. Each particle may slightly move around stationary position, and all system is rotated with the constant angular velocity. Set the perturbing body in circular orbit with radius  $r$  in ring plane. Let the radial perturbation of ring small  $x \ll R$  where  $R$  – radius of ring.

The condition of appearance of parametric resonance and related instable zones:

$$\frac{2\omega_0}{(\Omega - \omega_s)} = n, \quad n = 1, 2, 3, \dots \quad (4)$$

where  $\Omega$  and  $\omega_s$  – mean motions of perturbed and perturbing bodies accordingly. Here  $n$  – is an order of resonance.

The width of resonance is completely determined by satellite perturbation; it rapidly decreases with an resonance order increasing. For the first order by  $n$ :

$$\varepsilon_1 = \frac{n\omega_s^2 b_n}{2(n-2)\omega_0} \approx \frac{n\omega_s m / M b_n}{2(n-2)} \quad (8)$$

where  $b_n$  – Laplace coefficients, which easy calculated numerically.

In real planetary ring, perturbed by outer satellite, may appear parametric type of instability. In case neglected ring's particles gravity, result is in according with [3]. It is shown, that by taking into account interaction between ring's particles, it is possible to significantly change resonant structure in ring. The position of resonances is different than in case neglected particles gravity. There is one very important effect: the center of instability is shifted relative exact commensurability.

**The application to planetary rings:** The expansions of ring perturbation (radial and tangential) has a form [4]:

$$\begin{aligned}
U_r \equiv \frac{dU_{ring}}{dR} = & - \sum_j^{n-1} \frac{\gamma m_j}{R^2 (2 \sin(\psi_j/2))^2} + \\
& + \left( \sum_{j=1}^{n-1} \frac{3}{4} \frac{\gamma m_j}{R^3 \sin(\psi_j/2)} - \frac{\gamma m_j}{R^3 ((2 \sin(\psi_j/2))^3)} \right) x + \\
& \left( \sum_{j=1}^{n-1} \frac{3.75 \gamma m_j}{R^4 ((2 \sin(\psi_j/2))^3)} - \frac{15}{4} \frac{\gamma m_j}{R^4 ((8 \sin(\psi_j/2))^3)} \right) x^2 \\
& + O(x^3)
\end{aligned} \quad (5)$$

$$\begin{aligned}
U_l = & - \sum \frac{\gamma m R^2 \sin(\Psi)}{(2R^2 - 2R^2 \cos(\Psi))^{3/2}} + \\
& + \sum \left( \frac{3\gamma m (1 - \cos(\Psi)) R^2}{(2R^2 - 2R^2 \cos(\Psi))^{5/2}} - \frac{\gamma m}{(2R^2 - 2R^2 \cos(\Psi))^{3/2}} \right) \\
& \times R \sin(\Psi) x - \left( \sum \frac{3\gamma m (1 - \cos(\Psi)) R}{(2R^2 - 2R^2 \cos(\Psi))^{5/2}} - \right. \\
& \left. - \left( \frac{3\gamma m R}{(2R^2 - 2R^2 \cos(\Psi))^{5/2}} - \frac{15\gamma m (1 - \cos(\Psi))^2 R^2}{2(2R^2 - 2R^2 \cos(\Psi))^{7/2}} \right) \right) \\
& \times R \sin(\Psi) x^2 + O(x^3)
\end{aligned}$$

where  $\gamma$  - gravity constant,  $m_j$  - mass of ring particle,  $\Psi \equiv \psi_j = \frac{2\pi j}{N} + \phi_0$  - angular distance between  $j$ -particle and tested particle,  $\phi_0$  - small arbitrary angle.

In general case, it allow us to calculate centers of instabilities positions:

$$R^{3/2} = r^{3/2} \frac{n^2 - 2\sqrt{n^2 + (n^2 - 4)\alpha} r^3 / M}{n^2 - 4\alpha r^3 / M} \quad (6)$$

$$\alpha = \sum_{i=1}^{N-1} \frac{m_i}{(2R_0 \sin(\pi i / N))^3} \approx 1.258784 \rho \sigma^3 \quad (7)$$

Here  $\alpha$  - ring particles gravity perturbation,  $M$  - central mass,  $r$  - satellite orbit radius,  $\sigma = r_k N / (\pi R_0)$  - where  $\sigma$ ,  $\rho$ , and  $r_k$  - central configuration's «coefficient of filling», density and size of central configuration's particles respectively.

**Conclusions:** The resonance perturbation of planetary ring by distant satellite is considered. The gravity interaction between rings particles is taken into account. It is shown, that mutual interaction between particles of ring occur significant effect on resonance structure. The shift between simple mean motion resonances and parametric resonance zones is detected. This shift depends from ring properties. Unexpectedly, the resonance's structure depends only from particle's density, but not depends from particles (or ring) mass. The results applied to Saturn ring system. By varying ring's particles parameters, is possible to explain observed shift between actual ring distances and exact commensurability.

**References:** [1] Meyer K.R., Schmidt D.S. (1993). *Celestial mechanics and Dynamical Astronomy*, **55**, 289-303. [2] Szebehely, V. (1967). *Theory of Orbits. The Restricted Problem of Three Bodies*. Acad.press. New York & London. [3] Hadjidemetriou, J. D. (1985), *Proceedings of the Advanced Study Institute, Cortina d'Ampezzo, Italy, August 6-18, 1984* (A86-39351 18-90).. 213-225. [4] Rosaev A.E. (2003). *Proceedings Of Libration Point Orbits And Applications Conference, Parador d'Aiguablava, Girona, Spain 10 - 14 June, 2002* Eds. by G. Gomez, M. W. Lo and J.J. Masdemont, 623-637