

MODELING OF THE FREE LUNAR LIBRATION. N. Petrova¹, A. Gusev^{1,2}, ¹Kazan University, 18, Kremlyevskaja str., Kazan, 420008, Russia. ²National Astronomical Observatory, 2-12 Hoshigaoka, Mizusawa, Iwate 023-0861, Japan (E-mail: nk_petrova@mail.ru)

Introduction: Today the most interesting data on dynamics and internal structure of the Moon are already accumulated as a result of the different observations and space experiments. The Japanese space experiments Lunar_A and SELENE-missions, planned for 2006-2008 years, will contribute significantly to the information about the Moon: qualitative parameter Q, Love number k₂, core's radius R_c, core's density etc [2, 3, 5]. The searching for the free libration of the Moon and the study of this phenomenon [8, 9] may play an important role in determination of listed values in the Japanese experiments.

Three-layer Moon: Solidification of early planetary-scale lunar magma ocean [13] produced a chemically stratified mantle and core. According to lunar interior model [12], the core boundary pressure should be near 45 – 50 kbar and the temperature about 1000°C. The core cannot consist of pure iron under these circumstance because it would be difficult to keep an iron core from completely freezing, as cooling model suggest. Solidification of core begins from center. A small concentration of sulphur has been postulated to account for molten shell because sulphur is well known depress the freezing point of core alloy. Tidal heating in the inner core has also been demonstrated to help in keeping a liquid outer core shell. Calculation based on a more complete description of mantle convection and incorporating pressure and temperature dependent rheology [6, 11] suggest that the Moon cools mostly by thickening its lithosphere while the deep interior stays relatively hot.

Completely fluid or solid cores are end-members. A completely solid core cannot have been reached since there must be at least a thin fluid shell to apply the torque that LLR analyses detect. The relative sizes of the inner and outer cores would depend on initial S/Fe proportion and the present core temperature. For a core of radius of 350 km with a present temperature of 1400°–1700°K, and evolutionary cooling of 50°–150°, Stevenson D.J. and Yoder C.F [12] calculate a sulfur mass fraction of 0.04–0.13 and liquid shell thickness of 65–180 km.

It is known [4, 10] that for the two- or three layers Moon one (Free Core Nutation – FCN) or three (FCN, Free Inner Core Nutation – FICN and Inner Core Wobble – ICW) additional modes in a polar motion may be observed together with Chandler Wobble (CW), which were already observed by LLR for the Moon [7].

Lunar interiors and free nutations: Modeling was carried out with the purpose to detect a dependence of the free libration periods on various parameters characterizing the lunar core: core's radius, chemical composition, qualitative parameters Q. To do this the four cases were considered.

Case 1: Moon model composed of a rigid mantle and a liquid core of various densities: from eutectic

composition Fe-FeS (with different weight of sulphur) to pure iron core. The density of eutectic composition was varied from 4 gm/cm³ (high contents of S) up to 7 gm/cm³ (100% Fe). According to [14] the density of 5.3 gm/cm³ corresponds to 25% wt S. The restriction on radius of a core at the various densities was tested on the basis of mass relation of a core to whole Moon: it should not exceed 4-5 %.

Case 2: The same model as in Case 1, but the core is considered as a liquid iron with the density 7 gm/cm³. The dependence on the core ellipticity is modeled.

Case 3: The same model as in Case 1, only for the completely liquid iron core with the density 7 gm/cm³, including the dissipation in the core-mantle boundary (Figure 1). For the CW-frequency the formula is taken with term of second order, because in the first order there is no difference with Case 1.

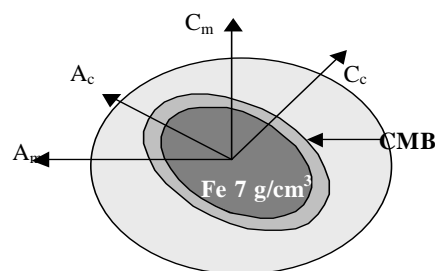


Figure 1. Scheme of the two-layer Moon with dissipation on CMB.

The dissipation coefficient R corresponds to the viscose damping at the core-mantle boundary without the effect of the electromagnetic coupling [1]. It depends on the Q-factor corresponding to FCN. Taking the various values of Q from LLR-analyses we have calculated set of R for different core's radiuses.

Case 4: Rigid mantle, fluid outer core (FOC) and solid inner core (SIC). The density of the FOC was taken 5.3 gm/cm³ (eutectic composition 25 wt % S and 75 wt % Fe) and density of the SIC – 7.7 gm/cm³ (solid iron). The modeling was performed with the purpose to detect a correlation of all four periods with the thick h of FOC. The thick h was taken as a part of a whole core and its values was varied from 20% to 60 % of R_c.

The following propositions were taken for the all four models:

- The lunar structure was taken as a *complex of homogeneous elliptical layers*: rigid mantle and liquid core or outer liquid/solid inner core.
- The parameters of the cores -- *size, ellipticity, densities and state of aggregation* – were taken in a vicinity of magnitudes obtained from LLR-analyses [14], [15].
- As any experimental information misses from a lunar inner core, the ellipticity of SIC e_s was taken

less on 5% than one of the FOC, as in the case for the Earth.

- The diapason of core's radius R_c was taken from 200 km to 600 km for all cases.

- Set of densities ρ_c for the Case 1: $4.5 \rightarrow 5 \rightarrow 5.5 \rightarrow 6 \rightarrow 6.5 \rightarrow 7$ (gm cm^{-3}).

- Set of dissipation parameter R for the Case 3: $R = 5 \times 10^{22} \rightarrow 1 \times 10^{23} \rightarrow 5 \times 10^{23} \rightarrow 1 \times 10^{24} \rightarrow 3 \times 10^{24}$ ($\text{erg} \cdot \text{sec}$)

Results:

- For the Case 1 the both periods (P_{CW} and P_{FCN}) depend very slowly on the density. Only for the radius great than 400 km the impact of density is remarkable: the difference in periods for eutectic composition with a density 5.5 gm/cm^3 with the pure iron core (7 gm/cm^3) is less than 0.04%. Because of this, in the Case 2 we consider only pure iron fluid core.

- The significant dependence on the core's ellipticity takes place for P_{FCN} : for the core, whose dynamical figure is similar to those of mantle ($e_c = 5.17 \times 10^{-4}$) the FCN-period is about 144 years, and the decreasing of the ellipticity only on 3% (5.0×10^{-4}) increases the period up to 149 years, i.e. on 3.5% too. The FCN-period for the ellipticity given by LLR-analyses (4×10^{-4}) is about 186 years. Therewith the correlation with the radius of a core is very weak.

At the same time the P_{CW} is weakly depends on the ellipticity and more essentially – on the core's radius.

- The CW-period does not feel the dissipation: this is an internal process for the whole Moon and it should not have an effect for these oscillations. But for the FCN the strong correlation with the dissipation is observed for core's radiuses R_c less than 400 km. According to the formula for FCN with dissipation two terms of opposite sign compete with each other, and due to this the extreme takes place. This fact is important indicator of internal processes, which may be derived from observations.

- In comparison with two-layer model for FCN and CW the three-layer model contributes no more 1% in the values of period for both kind of FOC.

- The magnitudes of periods for FICN and ICW were obtained for the first time. These values are sensitive to parameter characterizing the ellipticities of both cores and which is yet unknown with a certainty.

- The main tendency of behavior of two new periods is preliminary revealed: a) the FICN-period decreases both with the increasing of the core's radius and of the thick of fluid shell; b) conversely, the ICW-period increases with the increasing of the core's radius and of the thick of fluid shell.

The values of all periods for a liquid axial symmetric iron core with a radius of 350 km are given in the Table

R=350 km; $\rho=7 \text{ gm/cm}^3$, $e_c=5.17 \times 10^{-4}$					
	Case 1	Case 2 ($e_c=4 \cdot 10^{-4}$)	Case 3 (for the various dissipation parameter)	Case 4 (for the various inner core)	
CW	74.02	74.03	(R=5E22) 74.03	$R_s=280 \text{ km}$	74.08
			(R=3E24) 74.03	$R_s=175 \text{ km}$	74.02
FCN	144.02	186.88	(R=5E22) 144.58	$R_s=280 \text{ km}$	144.52
			(R=3E24) 144.22	$R_s=175 \text{ km}$	144.52
FICN				$R_s=280 \text{ km}$	634.22
				$R_s=175 \text{ km}$	515.90
ICW				$R_s=280 \text{ km}$	100.21
				$R_s=175 \text{ km}$	108.05

References: [1]. Getino J., Ferrandiz J.M. (1997) *Geophys.J.Int.*, 130, 326-334. [2] Gusev A., Petrova N., Kawano N. (2002) *IAA Transactions.*, 8, 83-85. [3] Gusev A., Kawano N., Petrova N. (2003) *Astron. & Astroph. Trans.*, 22, 141-147. [4] Gusev A., Kawano N., Petrova N. (2004) *Adv.Space Res* (in press) [5] Kawano N., Hanada H., Tsubokawa T., Iwata T. (2003) *Proc. of Int. Conf. "New Geometry of Nature"*, Aug.25–Sept.5, 2003, Kazan Univ. Press. 3, 114–119. [6] Konrad W., Spohn T. (1997) *Adv. Space Res.*, 19, 1511-1521. [7] Newhall X X, Williams J.G. (1997) *Cel. Mech. & Dyn.Astron.*, 66, 21-30. [8] Petrova N. (1996) *Earth, Moon and Planets*, 73, No 1, 71-99. [9] Petrova N., Gusev A.: (2001) *Cel. Mech. and Dyn. Astron.*, 80, Issue 3/4, 215-225. [10] Petrova N., Gusev A., Heki K., Hanada H., Kawano N. (2002) *IAA Transactions*, 8,143-145. [11] Spohn T., Konrad W., Breuer D., Ziethe R. (2001) *Icarus*, 149, 54-65. [12] Stevenson D.J., Yoder C.F. (1981) *LPS*, XII, 1043-

1044. [13] Warren P.H., Rasmussen K.L. (1987) *J.Geophys.Res*, 92, 3453-3465. [14] Williams J.G., Boggs D., Yoder Ch., Ratcliff J., Dickey J. (2001) *J Geoph. Res.*, 106, No E11, 27, 933-27, 968. [15] Williams J.G., Boggs D.H., Ratcliff J.T. (2004) *LPS*, XXXV. 1398.pdf.