SATURN'S ICY MOONS: A MODEL FOR THEIR ORIGIN AND BULK CHEMICAL COMPOSITION.

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Introduction: I report the preliminary results of a new model for the formation of Saturn's family of mid-sized icy moons, namely Mimas, Enceladus, Tethys, Dione, Rhea and Iapetus. It is proposed that these 6 moons condensed from a concentric family of orbiting gas rings that were shed equatorially by the contracting proto-Saturnian (hereafter p-Sat) cloud. Gas ring shedding is a central feature of the Modern Laplacian Theory of Solar system origin (hereafter MLT) [1,2]. In the case of the p-Sat cloud, the rings are shed at orbital radii R_n (n = 0,1,2,..) that lie close to the present mean orbital distances of the satellites, except for Iapetus. It is suggested that this moon initially formed close to the present mean distance of Titan, namely $R_{Ti} = 20.27R_{Sat}$, where R_{Sat} = 60,268 km is Saturn's equatorial radius. Iapetus was then displaced dynamically to its present orbit as a result of Titan's capture by the p-Sat cloud.

Now Titan's mass exceeds that of the next largest moon Rhea by a factor of ~ 58. It is also ~14 times larger than the expected mass of condensate $m_{cond} \approx 9.4 \times 10^{24}$ g which can form from a p-Sat gas ring – see below. On this basis, it has been argued that Titan cannot be a native moon of Saturn [3,4]. Instead, it originally condensed as a secondary growing embryo within the gas ring that was shed by the protosolar cloud (hereafter PSC) at Saturn's orbit. It was then later captured through the action of gas drag at the edge of the p-Sat cloud, when the radius of that cloud was ~20 R_{Sat}.

If the above picture is correct, then Titan's capture was secured long before the p-Sat cloud had shrunk to the present orbit of Rhea ($8.75R_{Sat}$). Perhaps Titan gravitationally stymied the formation of moons from the gas rings that were shed at radii ~$11.5R_{Sat}$ and ~$11.5R_{Sat}$, in much the same way that Jupiter thwarted the growth of a single planet at the asteroidal belt.

The Modern Laplacian Theory: According to the MLT, the planetary system and regular satellite systems of the gas giant planets condensed from concentric families of orbiting gas rings. These rings are shed by the parent cloud as a means for disposing of excess spin angular momentum during gravitational contraction. The mean orbital radii R_n and masses m_n (n = 0,1,2,..) of the rings satisfy the equations $R_n/R_{n-1} = (1 + m_n/M_n f_n)^2$. Here M_n, f_n denote the residual mass and moment-of inertia factor of the cloud after shedding the n^{th} ring. If the cloud contracts uniformly, so that both f_n and m_n/M_n stay constant, then so also is R_n/R_{n-1} a constant.

If we apply the above equations to the PSC and set $M_n \approx M_\odot$, $f_n \approx 0.01$, we find $M_n \approx 6.0 \times 10^{30}$ g. Next using recommended protosolar elemental abundances [5] and sequestering all rock-like elements into oxides and sulphides and C,N,O as hydrides, the total condensable mass of rock and ice in each ring is $m_{cond} \approx 15M_\oplus$. This agrees with the observed masses of Uranus and Neptune and with the observed core mass of Saturn [6]. Next applying the same equations to the p-Sat cloud, with $f_n = 0.01$, $M_n \approx M_{Sat} = 5.685 \times 10^{29}$ g and $\langle R_n/R_{n-1}\rangle = 1.30$, which is the observed mean distance ratio from Mimas to Rhea, we obtain $m_n \approx 8.0 \times 10^{26}$ g. The expected mass of rock + ice (H_2O & NH_3) condensate in the gas ring at Rhea's orbit is then $m_{cond} \approx 9.4 \times 10^{24}$ g. But the observed mass of Rhea is 2.32×10^{24} g. This suggests that the process of accumulation of solids within the proto-solar gas ring at Saturn's orbit was so efficient that the gaseous envelope (consisting mostly of H_2 & He) that was later captured gravitationally by the core to form the p-Sat cloud was heavily depleted in its heavy element fraction Z by a factor of ~4. That is, $Z_{Sat} \approx 0.25Z_\odot$.

A New Protosolar Cloud Model: In order to determine the bulk chemical compostion of the native icy moons of Saturn, it is first necessary to construct a numerical model for the PSC. This calculation yields the thermochemical and compositional state of the gas which makes up the p-Sat cloud. Now in order for the contracting PSC to dispose of its excess spin angular momentum in discrete gas rings, it is necessary that the interior of the cloud be pervaded by a large radial turbulent stress p_{turb} arising from strongly supersonic, thermal convective motions [7]. This stress is given by $p_{turb} = \beta(r)\rho GM(r)/r$. Here $\rho = \rho(r)$ is the local gas density, $M(r)$ is the mass interior to radius r and $\beta = \beta(r)$ is the turbulence parameter. The total pressure at each point is $p_{turb} + p_{gas}$, where $p_{gas} = \rho\Re T/\mu$ is the usual gas pressure, T is the temperature and μ is the mean molecular weight.

Each non-rotating model of given surface radius R_s and total mass $M_s = M(R_s)$ has an adiabatic core of radius r_0 inside which $\beta = \beta_0$, a constant. The surface of the cloud is defined where the dimensionless temperature function $\theta \equiv \mu_c T(r)/\mu T_c = \theta_s$, a constant, and c refers to the centre. The core itself consists of an inner zone of radius r_1 in which all H is taken to be H_1 (or H^+) and an outer zone in which it is all H_2. Lastly, the core is surrounded by a superadiabatic envelope of polytropic index $n_t = -1$ in which β falls to 0 according as $\beta = \beta_0(\theta - \theta_s)/(\theta_0 - \theta_s)$, where $\theta_0 = \mu_c T(r_0)/\mu_0 T_c$.

Rotation is included using the atmospheric approximation [1]. If the controlling parameters β_0, θ_0, θ_s and n_t stay constant during the contraction, the PSC sheds gas rings whose mean orbital radii R_n (n = 0,1,2,..) form a nearly geometric sequence. The initial cloud mass M_i is chosen so that the final cloud mass is $M_\odot$ = 1.98892×10^{33} g. Setting θ_s = 0.00227185, $F_s \equiv \theta_0/\theta_s$ = 8.5338, and β_0 = 0.120566 ensures (i) that the mean orbital spacings of the gas rings from Jupiter to Mercury matches the observed planetary spacings, (ii) that the condensate bulk density ρ_{cond} at Mercury's orbit results in a planet of Mercury's physical size whose mean density equals the observed value, namely 5.43 ± 0.01 g/cm^3 [8], and (iii) that only a fraction ϕ_{H2O} = 0.700 of the total H_2O at Jupiter's orbit condenses out. This ensures a composition for the proto-Jovian cloud which can account for the observed 0.55:0.45 rock-to-ice mass ratios in Ganymede and Callisto [3].

At Saturn's orbit, where the PSC gas ring temperature is T_n = 94 K, and the mean orbit pressure is p_n = 4.9 × 10^{-7} bar, the condensate consists of rock (mass fraction 0.4923), water ice (0.4739) and graphite (0.0338). The rock is almost anhydrous and has density 3.668 g/cm^3. The total fraction of the water vapour that is condensed is ϕ_{H2O} = 0.974. The condensate mean density at the present-day black body temperature at Saturn (T_{Sat} = 76 K) is ρ_{cond} = 1.5225 g/cm^3.

Results and Predicted Chemical Compositions: We now compute the contraction of the p-Sat cloud. The parameter $\theta_{s,Sat}$ = 0.0043318 is chosen so that $\langle R_n/R_{n-1}\rangle_{Sat} = 1.30$. The parameter $\beta_{0,Sat}$ is normalized against the solar value $\beta_{0,\odot}$ assuming the linear dependence $\beta_0 \propto (F_s - 1)$. This leaves F_s as a free parameter. F_s controls the absolute scaling of the gas ring temperature distribution. We also define W_{H2O} to be the ratio of the H_2O content of the envelope relative to the solar value. Initially, $W_{H2O} = Z_{Sat}/Z_\odot$ = 0.25. Table 1 lists the present satellite orbital radius in units of R_{Sat}, the gas ring mean orbit pressure p_n and temperature T_n for the case F_s =5.6 T_{H2O} is the condensation temperature of water ice on the mean orbit of each ring. The last column gives the mean density ρ_{cond}. This is calculated for temperature T_{Sat} = 76 K.

The current observed satellite mean densities ρ_{obs} are given in the last column of Table 2 (R.A. Jacobson, priv. comm.). Comparing the values of ρ_{cond} in Table 1 with ρ_{obs}, the agreement is satisfactory only for Rhea and Dione. Iapetus (and Hyperion – which condensed at ~25 R_{Sat}) are the only moons to contain CH_4. This is tied up with the H_2O as the clathrate hydrate CH_4•5.75H_2O. It has mass fraction 0.057. The predicted composition for Tethys is far too rocky and dense. The mass rock fraction here is $X_{rock,Te}$ = 0.500.

Table 1: Gas ring properties for the p-Saturn cloud

Moon	R_n/R_{Sat}	p_n/bar	T_n/K	T_{H2O}/K	ρ_{cond}
Mimas	3.080	27.1	318	272	3.16
Enceladus	3.951	10.3	250	260	2.13
Tethys	4.890	4.49	204	251	1.44
Dione	6.626	1.38	153	239	1.43
Rhea	8.746	0.468	120	230	1.22
Iapetus	20.00	0.0188	74	205	1.29

Table 2: Condensate composition and mean density

Moon	W_{H2O}	X_{H2O}	X_{NH3}	ρ_{cond}	ρ_{obs}
Mimas	3.0	0.	0.	3.15	1.16±0.01
Enceladus	3.0	0.900	0.	1.00	1.8±0.2
Tethys	3.0	0.914	0.	0.99	0.99±0.01
Dione	0.25	0.506	0.	1.43	1.49±0.04
Rhea	0.25	0.384	0.240	1.22	1.23±0.04
Iapetus	0.25	0.370	0.239	1.13	1.11±0.04

We now advance the idea that once the p-Sat cloud had shrunk beneath the orbit of Dione, the central planetary core of mass $M_{core} \sim 15M_\oplus$ releases a substantial quantity of its volatile component into the envelope of mass ~ 80$M_\oplus$. If some 16% of the core H_2O mass is released, the value of W_{H2O} rises from 0.25 to 3.0. That is, the water content of the envelope is now 3 times the solar value. Looking at Table 2, which gives the mass fractions X_i of H_2O, NH_3 and CH_4 at each orbit, we see that the agreement of ρ_{cond} with ρ_{obs} is now excellent for all moons except Enceladus and Mimas. For Iapetus, it is assumed that later thermal evolution releases the CH_4 from the clathrate structure. It is interesting that the predicted density for Enceladus, namely 1.00 g/cm^3, coincides with the value derived from Voyager-based studies of the shape of this moon [9]. The Cassini Orbiter will soon obtain a direct measurement of the Enceladus mean density.

References: [1] Prentice A.J.R. (1978) *Moon Planets*, 19, 341–398. [2] Prentice A.J.R. (2001) *Earth, Moon & Planets*, 87, 11–55. [3] Prentice A.J.R. (1984) *Earth, Moon & Planets*, 30, 209. [4] Prentice A.J.R. (2004) *BAAS*, 36, 1116. [5] Lodders K. (2003) *ApJ*, 591, 1220. [6] Anderson J.D. et al. (1980) *Sci*, 207, 449. [7] Prentice A.J.R. and Dyt C.P. (2003) *MNRAS.*, 341, 644–656. [8] Anderson J.D. et al. (1987) *Icarus*, 71, 337–349. [9] Dermott S.F. and Thomas P.C. (1994) Icarus, 109, 241–257.