SHAPE, SPIN, AND THE STRUCTURE OF ASTEROIDS, CENTAURS, AND KUIPER BELT OBJECTS.

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Introduction: Holsapple [1,2] explored the relationship between the shape and spin state of a triaxial ellipsoid as a function of its density, including the role of internal strength in maintaining certain body shapes against the resulting stresses.

The possibilities for such a theory are tantalizing; given a shape model and a rotation rate derived from observations of light curves [3] one might put limits on the density and strength of the body. But the application of this theory to actual solar system bodies has been limited by a number of issues:

(1) In the past ten years, images of asteroids and comets from radar, spacecraft, and AO observations have demonstrated that a triaxial ellipsoid is at best a crude approximation of such bodies' true shapes.

(2) To derive a complete shape model requires observations of light curves over a large fraction of the body's orbit: tedious for asteroids, challenging for Centaurs, and impossible within a human lifetime for Kuiper Belt Objects (KBOs). Lacking that, the best that a given light curve can supply is a lower limit on the ratio of only two of the three dimensions of the ellipsoid (a/b, presuming rotation about the shortest axis c). But rotation-spin theory relates spin/density to a/c, not a/b (as a weak function of b/c). Applying this theory to comet nuclei is even more problematic, since they are subject to splitting and changes in spin state due to their activity. For example, Comet Halley does not spin about a principle axis [4].

(3) Given a finite internal strength, the problem is no longer a simple relationship between spin, shape, and density. There is no unique density or stress state for a given shape and rotation rate.

Given these difficulties, we can at best only put limits on the internal structure and shape of such objects. Here we take three approaches: to examine how the theory works for those bodies (mostly asteroids) with already known shapes and densities; to note where the direction of the uncertainties drives the data; and to draw conclusions from trends of data from many bodies, rather than attempt to make strong statements about any given body.

Asteroid Shape and Strenth Holsapple [2] produced curves of constant internal friction angle, a measure of internal strength, on a plot of c/a vs. a normalized spin rate $\Omega = \omega/\sqrt{(\pi G \rho)}$ and on this plot indicated the position of nearly 1000 asteroids. The asteroid data essentially fills the entire space of the plot, including a few objects that are spinning so rapidly that they must have significant shear strength.

In order to plot these objects he had to make reasonable assumptions about their shapes (b = c) and densities (1.5 g/cm^3 for C type asteroids, 8.0 g/cm^3 for M types, and 3.0 g/cm^3 for S and "other" types). However, in recent years we have been able to determine independently the actual density and triaxial shapes for a number of small solar system objects, including asteroids, small moons, and comets. Plotting these objects on the Holsapple graph (Figure 1) gives a revealing picture of these bodies' internal state.

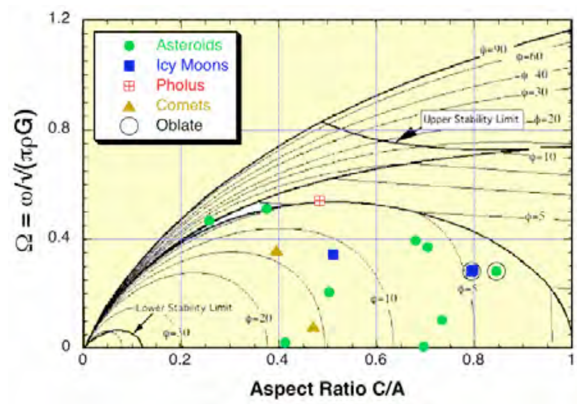

Figure 1: Implied internal strength for well-characterized bodies. Adapted from Holsapple [1]

Holsapple plots two sorts of curves here. The heavy lines demark stress stability limits, while lighter lines are the contours of internal friction angle. Most notable here is the heavy half-oval that arcs from (0,0) to (1,0); this is a curve indicating where the stresses $\sigma_x=\sigma_z<\sigma_y$ and below it, $\sigma_x<\sigma_z<\sigma_y$. Note that *all* the well-characterized bodies lie in this region or on its boundary. (In fact, an inspection of the original graph in [2] suggests that the majority of all asteroids may plot in this region.) The significance of this stress state is that the internal friction of the body is essentially maintaining the elongated shape of the body against gravity, rather than against spin. Note also that all bodies require less than a 20° internal friction angle, and many less than 10°, significantly less than the canonical 30° usually assumed for geological materials. They may be weak.

Minor axes The Holsapple curves used here are those calculated by assuming that the shorter axes b and c are equal. In reality there is always some inequality; indeed, for the two points circled, b is closer to a than c and these bodies might be better described as oblate than prolate in shape. For the other bodies, is b=c a good approximation? Looking at asteroids, we can see in Figure 2 that c/b is greater than 0.9 for most imaged asteroids, especially for asteroids whose longest axis is greater than ~50 km.

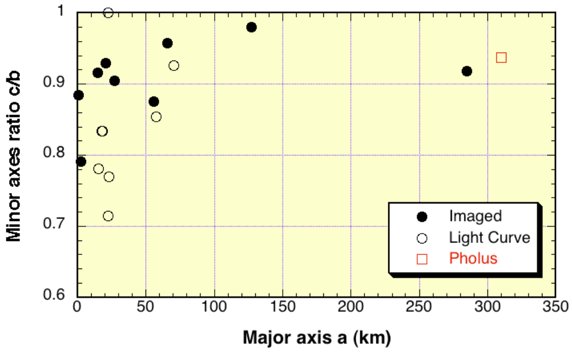

Figure 2: For radii >50 km, c/b >0.9. Filled circles are imaged asteroids; open symbols are prolate objects modeled from light curves only.

This trend is somewhat masked when comet nuclei and icy satellites are included, because many of them already have b approaching c even at very small sizes. For large objects such as KBOs b ≈ c may be a very good approximation; note that Pholus, marked in red, follows the asteroid trend. In any event, these objects are *not* Jacobi ellipsoids.

Spin vs. density That so many asteroids lie in the lower part of Figure 1, i.e. with similarly low values of Ω ($\propto \omega/\sqrt{\rho}$) despite their large range of spin rates, suggests that the faster spinning bodies may also be denser. A plot of density vs. spin rate ω is given in Figure 3: as can be seen, there does seem to be a correlation. By eye one can infer two parallel lines in the data, but given the few data points plotted, and that there does not seem to be any systematic difference between bodies on either line (e.g. Phobos and Deimos are on different lines), it is not clear that there is any significance to these two "lines."

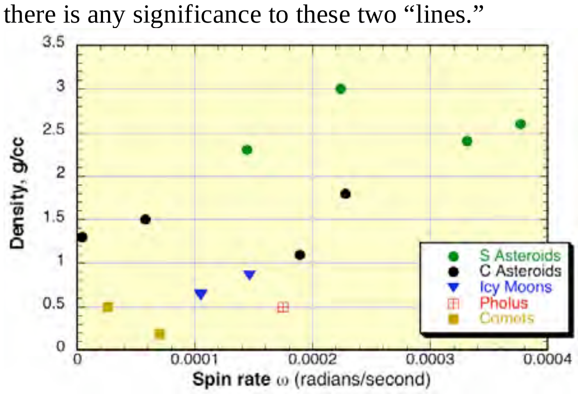

Figure 3: Density and spin may be related.

Centaurs and KBOs These data on asteroids suggests that KBOs may be approximated as prolate bodies with c=0.95 b, and may lie near the $\sigma_x=\sigma_z<\sigma_y$ line on Figure 1. Plotting lightcurves and spin periods for 30 Centaurs and KBOs [5, 6] we see:

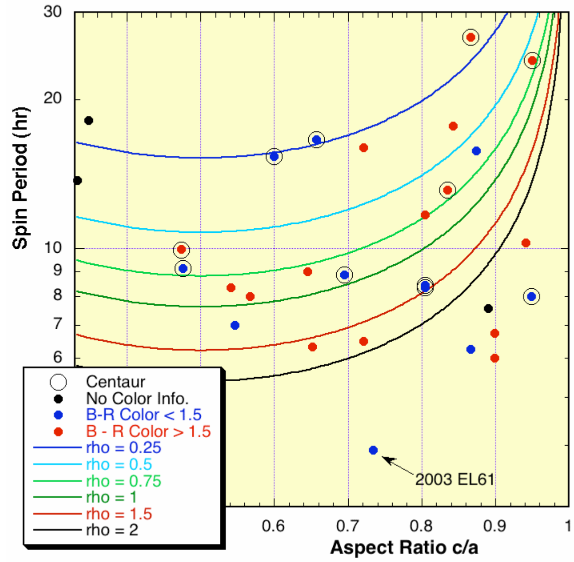

Figure 4: Centaurs and KBO spins vs. c/a (from light curve amplitudes), with calculated density contours.

The density contours are found by assuming the $\sigma_x=\sigma_z<\sigma_y$ relationship of Figure 1 and solving for ρ as a function of c/a and ω. For bodies below that curve, these densities are *lower limits*. We find no correlation between spin rate or light curve amplitude and any orbital or physical parameter, including size or color, except that the very large object 2003 EL61 must be much denser than the typical KBO.

The plotted aspect ratio points are calculated from observed light curves; all (except for Pholus) are likely underestimates of c/a — they could be shifted to the left. For large c/a bodies, this implies lower densities. Once the c/a ratio is less than 0.7, however, moving the points to the left does not significantly shift the inferred density of the object.

Not surprisingly, most inferred KBO densities lie between 0.25 and 1 g/cm^3. Using the average spin rate of the 29 KBOs and dropping the average c/a by a factor of cos 45° one derives a "mean density" of 0.45 g/cm^3. This is similar to the mean densities reported for comets Borrelly and Tempel 1 [5].

References [1] Holsapple, K. A. (2001), *Icarus* **154**, 432-448. [2] Holsapple, K. A. (2004), *Icarus* **172**, 272-303. [3] Magnusson, P. (1986), *Icarus* **68**, 1-39. [4] Belton et al. (1991), *Icarus* **93**, 183-193. [5] Britt et al., this conf.; Tegler et al. (2005), *Icarus* **175**, 390-396. [6] Harris, A. W. and Warner, B. D. (2005), http://cfa-www.harvard.edu/iau/lists/LightcurveDat.html