

## NON-CHAOTIC OBLIQUITY VARIATIONS OF MARS

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**Introduction:** In the absence of energy dissipation, obliquity variations of Mars would likely be chaotic. However, relatively small amounts of dissipation suffice to suppress the chaotic variations. Tidal dissipation within Mars, as constrained by the observed evolution of the orbit of Phobos, appears sufficient to regularize the obliquity variations. If Mars has a fluid core, viscous core-mantle coupling would also tend to damp the obliquity variations. An adequate rate of dissipative damping is a necessary, but not sufficient, condition to establish that the obliquity variations are fully damped. Evidence in support of this stronger conclusion comes from two further observations. First is that the current orientation of the spin pole of Mars (in both obliquity and azimuth) is close to that predicted for a fully damped spin state. Second is that numerical integrations of the equations of motion for the spin pole, over an interval of a few million years, when performed both with and without dissipation, differ only slightly, suggesting that the spin pole has been driven to the fully damped state. It thus appears that the free obliquity of Mars is small, and that dissipation plays an important role in the rotational dynamics of this body, not unlike the situation for Mercury and Venus.

**Obliquity without dissipation:** It has been claimed that the obliquity variations of Mars are chaotic in nature [1,2,3], and this claim has been widely embraced by the Mars paleoclimate research community. We first discuss how the system would behave without any dissipative processes, and then consider how dissipation changes that state.

The obliquity of Mars is, of course, just the angular separation between the spin pole  $\mathbf{s}$  and the orbit pole  $\mathbf{n}$ . These two unit vectors determine the relative orientations of the orbit plane and the equator plane, and this orientation is the dominant influence on the seasonal and latitudinal pattern of insolation [4,5,6]. The obliquity is given by

$$\cos[\varepsilon] = \mathbf{n} \cdot \mathbf{s} \quad (1)$$

and its current value is [7,8]

$$\varepsilon = 25.1894 \text{ deg} \quad (2)$$

The evolution of the spin pole, in the absence of dissipative effects, is governed by the non-linear differential equation which equates change in spin angular momentum to applied solar torque. It can be written in the compact form [9,10]

$$\frac{ds}{dt} = \frac{\alpha}{(1-e^2)^{3/2}} (\mathbf{n} \cdot \mathbf{s})(\mathbf{s} \times \mathbf{n}) \quad (3)$$

with  $e$  the orbital eccentricity, and  $\alpha$  a rate parameter which relates the mass distribution within Mars to the solar torque. It is given by

$$\alpha = \frac{3}{2} \frac{n^2}{\sigma} \left( \frac{C - (A + B)/2}{C} \right) \quad (4)$$

where  $n$  is the orbital mean motion,  $\sigma$  is the spin rate, and the principal moments of inertia are  $A < B < C$ . For Mars, the observed spin pole precession rate is [7,8]

$$\frac{\alpha (\mathbf{n} \cdot \mathbf{s})}{(1-e^2)^{3/2}} = (7.576 \pm 0.035) \text{ arc sec/yr} \quad (5)$$

and the parameter  $\alpha$  has a corresponding current value of

$$\alpha = (8.263 \pm 0.038) \text{ arc sec/yr} \dots \quad (6)$$

If the orbit pole, orbital eccentricity, and oblateness of the figure of Mars were all constants, then the solar torque acting on the oblate figure of Mars would cause the spin pole to precess about the orbit pole at a uniform angular rate, and at fixed angular separation. In that case the obliquity is constant, and the motion is just that of a free (unforced and undamped) spherical pendulum.

If there are variations in the orientation of the orbit pole, the spin pole still attempts to precess about the instantaneous position of the orbit pole, but the obliquity is generally no longer constant. If the motion of the orbit pole is slow enough that the spin pole can follow it, then the obliquity will remain nearly constant. If, on the other hand, the orbit pole moves more rapidly than the spin pole can follow, the obliquity variations will very nearly reflect the orbit pole motion. The most dramatic variations in obliquity occur when the orbit pole is forcing the spin pole at its resonant frequency. Variations in orbital eccentricity and/or gravitational oblateness of Mars will further modulate the obliquity variations.

The primary reason for vigorous variations in obliquity of Mars is that its spin pole precesses at a rate which is close to resonance with the rates of motion of the orbit pole. In contrast, the Earth's spin pole precession rate, due to proximity of the Moon, is much faster than any of its orbital variations, and the corresponding obliquity variations are quite mild

**Obliquity energetics:** The spin configuration with lowest potential energy is that in which the spin pole and orbit pole are parallel. The difference in potential energy between that configuration and the present is

$$\Delta E = \frac{3}{4} n^2 (C - (A + B) / 2) (1 - \cos[\varepsilon])^2 \quad (7)$$

$$= 2.21 \cdot 10^{19} J$$

Tides raised on Mars by the Sun will dissipate energy and tend to damp the obliquity. The rate of tidal energy dissipation for a non-resonant rotator, in a circular orbit and with zero obliquity provides a lower bound for Mars, and can be written as [11]

$$\frac{dE}{dt} = \frac{3}{2} \left( \frac{k_2}{Q} \right) \frac{(\sigma - n) n^4 R^5}{G} \quad (8)$$

$$= 9.26 \times 10^{10} W$$

where  $k_2$  is the degree two Love number,  $Q$  is the tidal quality factor,  $R$  is the radius of the body, and  $G$  is the gravitational constant. In the estimate above the estimate of  $Q$  is derived from recent analysis of the orbital decay of Phobos [12], which samples Mars at a frequency close to that of solar tides. If all of the tidal dissipation went into damping the obliquity, it would be gone in only a decade. However, most of the dissipation goes into slowing the rotation of Mars.

**Obliquity with dissipation:** If the orbit pole were precessing at a uniform rate, the expected outcome of dissipative processes would be to drive the spin pole into a Cassini state [13,14,15]. In such a state, the spin pole, the orbit pole, and the invariable pole, about which the orbit pole is precessing, all remain coplanar. For that to occur, the spin pole adjusts the length of its path about the orbit pole so that, though the angular rates of the two moving poles need not match, their periods of motion do match. The Moon occupies such a state, as was noted by G.D. Cassini in 1693, though the dynamical basis of that configuration was not understood until much later.

If the orbit pole is precessing at a non-uniform rate, as is the case for Mars, then no such Cassini state is possible. However, if the motions of the orbit pole and spin pole are represented by Poisson series, and dissipation is included in the analysis it emerges that, on a mode-by-mode basis, the spin and orbit pole motions are again able to be synchronized. This type of analysis has previously been applied to Venus [16,17], Mercury [18], and the Galilean satellites [19].

When it is applied to Mars, we find that the spin pole orientation, both obliquity and azimuth, agree with observation. We thus conclude that the free obliquity has been damped and that the obliquity variations are not chaotic.

**Implications:** One of the diagnostic features of chaotic dynamical systems is that initially nearby configurations evolve along trajectories which exponentially diverge, either forward or backward in time. It is informative to compare this behavior with that seen in Hamiltonian and dissipative systems. In a Hamiltonian

system, Liouville's volume theorem states that the (properly defined) volume of a region in phase space remains constant under the flow of the system. Dissipative systems, in contrast, contract the volume of phase space along the flow. The role of dissipation in regularizing otherwise chaotic systems is an area of active research [20,21], and it is difficult, at this juncture, to make useful generalizations.

However, it appears that the system with dissipation will follow trajectories qualitatively similar to those estimated without dissipation. Behavior of the spin pole at times in the far distant past are equally inaccessible either way. Rather ironically, dissipative solutions allow better predictions of future behavior, as small errors in initial conditions damp out, but all of the useful information in the geologic record pertains to the past.

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