FORCED OBLIQUITY VARIATIONS FOR THE MAJOR SATELLITES OF SATURN.

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Introduction: We present a model for temporal variations in the obliquity, or angular separation between spin and orbit poles, for the 7 most massive satellites of Saturn, which are Mimas, Enceladus, Tethys, Dione, Rhea, Titan, and Iapetus. Our motivation in doing this has two primary aspects. First is that the tidally damped orientations of the spin poles depend on the moments of inertia of the satellites, and thus, if the obliquities can be measured accurately enough, they will provide important information about internal structure. Second is that a finite obliquity adds to the rate of energy dissipation associated with the tide raised by Saturn. If the forced obliquities are large enough, they can provide an additional source of internal heating for these bodies.

We assume that tidal dissipation has, in each case, driven these bodies to a fully damped rotational state, in which any memory of initial conditions has been erased. If the precession rates of the orbit poles were steady, as is nearly the case for Titan, then the expected spin state would be very much like that of the Moon, with the spin pole, orbit pole, and invariable pole (very nearly the spin pole of Saturn) remaining coplanar, as the spin pole precesses about the orbit pole, and the orbit pole precesses about the invariable pole. Such co-planar configurations are known as generalized Cassini states [1,2], as they generalize an aspect of lunar dynamics first commented upon by G.D. Cassini.

Orbit Variations: In the Saturn system, each of the satellite orbit planes is subject to torques from the oblate figure of Saturn, the Sun, and each of the other satellites, and as a result, the orbit planes precess at non-uniform rates. The expected spin pole behavior, in that case, is more complicated than for a uniformly precessing orbit. However, on a mode-by-mode basis, the co-planarity still remains [3,4]. That is, if the motions of the spin and orbit pole are each represented as a Poisson series in time, then each term in the series expansion will follow the simple pattern seen for uniform precession.

The dominant torque on each orbit is from the oblate figure of Saturn, and that makes the orbits principally precess about Saturn's spin pole. A solar torque tries to make the satellite orbits precess about Saturn's orbit pole. The Laplacian plane, in which these two contributions are balanced, is close to Saturn's equator for small orbits, but is close to Saturn's orbit plane for the more distant satellites [5,6]. The configuration of lowest potential energy for a pair of circular orbits is that in which they are coplanar. Each pair of satellite orbits thus exerts torques which attempt to achieve the state of lowest energy. However, due to inertial effects, the result is an exchange of angular momentum between the pair, and co-precession about their net angular momentum direction. When all of the secular torques are accounted for, a system of N coupled satellite orbits (about an oblate primary) precesses with N independent modes of oscillation. Each of the orbits participates in each of the modes of oscillation, but with varying amplitudes. In the Saturn system there are several mean motion resonances, which complicates the torque balance [7]. However, that mainly influences evolution of the eccentricities, which are small enough to ignore in this initial exploration.

We have constructed a secular variation model for the inclinations and nodal longitudes of the 7 largest satellites in the Saturn system, and include both the oblate figure of Saturn, and the Sun (as a distant but massive satellite). The normal mode oscillation periods are {0.98, 2.36, 4.98, 11.7, 35.8, 699, 2923) years. Each of the satellite orbits has a response at each of these periods, but the dominant response of the j-th satellite is at the j-th period. For example, Titan's inclination mainly varies with a 700 year period.

Spin Variations: The dominant torque on the spin of each satellite is that from Saturn. It depends on the distance and mass of Saturn, the relative orientations of the spin and orbit poles, and on the principal moments of inertia (A < B < C) of the satellite. The orientations of the spin and orbit poles are specified by unit vectors $\hat{s}$ and $\hat{n}$, respectively. Variations in the orientation of the spin pole are governed by the torque balance, which can be written as [3,4]

$$\frac{d\hat{s}}{dt} = \left(\alpha\,(\hat{n}\cdot\hat{s}) + \beta\right)\left(\hat{s}\times\hat{n}\right)$$

where the precession rate parameters are

$$\alpha = p + q \; and \;\; \beta = -q$$

with

$$p = \frac{3n}{2}\left(\frac{C - (A+B)/2}{C}\right)\left(1 + \frac{3}{2}e^2 + ...\right)$$

$$q = \frac{3n}{8}\left(\frac{B - A}{C}\right)\left(1 - \frac{5}{2}e^2 + ...\right)$$

where e is orbital eccentricity, and n is orbital mean motion (mean angular rate). For small inclinations, the effective spin pole precession rate is just p+q. The obliquity is simply given by

$$\cos[\varepsilon] = \hat{n} \cdot \hat{s}$$

As is evident from these equations, it is important to know the moments of inertia of the satellites. However, at present, they not well known. For Titan we assume a hydrostatic response to tidal and rotational influences [8, 9], and assume that the mean moment I=(A+B+C)/3 is in the range $0.3 \leq I/MR^2 \leq 0.4$. For the others, we simply estimate the moments from the observed shapes [10] and an assumption of uniform density. The corresponding spin pole precession period estimates are {0.031, 0.11, 0.22, 1.34, 3.41, 60-90, 3.13} years, with the range for Titan corresponding to the assumed range of mean moments. We note that, with the exception of Iapetus, the spin pole precession rates are 10-30 times faster than the corresponding orbit pole precession rates. For Iapetus, the ratio is close to 10^3 due to the non-spherical shape (and correspondingly large torques) of that body. As a result, the obliquity values are typically 10-30 times smaller than the inclination values, with the exception of Iapetus.

The secular variation model represents the projections of the spin and orbit poles onto the invariable plane as a sum of complex exponentials. The maximum possible value of the angle is obtained by summing the amplitudes. The minimum possible value is either zero or the difference between the largest amplitude and the sum of all the others, if that difference is positive. In the table we list the current, minimum, and maximum values of the inclinations, obliquities, and eccentricities. The inclinations are referred to Saturn's equator plane, rather than the local Laplacian plane.

Tidal energy dissipation is a potentially important heat source in these satellites. It arises from frictional effects associated with motion of the tidal bulge raised on these bodies by Saturn. The rate of tidal energy dissipation is given by [11, 12]

$$\frac{dE}{dt} = -\frac{5n^5R^3}{4\pi G}\left(\frac{k}{Q}\right)\left(7e^2 + \sin[\varepsilon]^2\right)$$

where G is the gravitational constant, k is a tidal Love number, and Q is a loss factor. Our estimates of forced obliquity values for these satellites suggest that the obliquity tides do not contribute significantly to the total dissipation rate, and that eccentricity tides dominate.

The best prospect for measuring these satellite obliquities is presented by Titan. Matching of Cassini radar images, in areas of overlap, should yield an estimate of the spin pole orientation with an ultimate accuracy of order 0.01 degree. That will yield an estimate of the polar moment of inertia, with an accuracy of a few percent. If measured obliquity values for these bodies, when they become available, are appreciably outside the predicted range, it would strongly suggest that either tidal dissipation has been surprisingly weak, or that a relatively recent excitation event has occurred.

References: [1] Colombo G. (1966) AJ, 71, 891-896. [2] Peale S.J. (1969), AJ, 74, 483-489. [3] Bills B.G. (2005) JGR, 110,E04006. [4] Bills B.G. (2005) Icarus, 175, 233-247. [5] Ward W.R. (1981) Icarus, 46, 97-107. [6] Ward W.R. and Hamilton DP, (2004) AJ, 128, 2501-2509. [7] Christou A.A. and Murray C.D. (1997) AA, 326, 416-427. [8] Hubbard W.B. and Anderson J.D. (1978) Icarus, 33, 336-341. [9] Rappaport N. et al. (1997) Icarus, 126, 313-323. [10] Thomas P.C. et al (2006), LPS 37, abs 1639. [11] Wisdom, J. (2004) AJ, 128, 484-491. [12] Winn J.N. and Holman M.J. (2005) ApJL, 628, 159-162.

Table 1. satellite orbit and spin parameters

body	inclination (deg)			obliquity (deg)			eccentricity (%)		
	current	min	max	current	min	max	current	min	max
Mimas	1.572	1.571	1.573	0.0505	0.0505	0.0505	1.960	1.960	1.961
Enceladus	0.009	0.006	0.018	0.0005	0.0005	0.0007	0.470	0.468	0.470
Tethys	1.091	1.088	1.097	0.0517	0.0517	0.0517	0.010	0.006	0.014
Dione	0.028	0.000	0.038	0.0036	0.0005	0.0036	0.220	0.197	0.222
Rhea	0.362	0.255	0.426	0.0362	0.0350	0.0365	0.100	0.016	0.212
Titan	0.317	0.278	1.197	0.0481	0.0472	0.0494	2.880	2.876	2.883
Iapetus	15.142	6.034	23.815	0.0094	0.0093	0.0097	2.830	2.686	2.980