TIDAL DISRUPTIONS II: SOLID BODIES WITH ALL KINDS OF STRENGTH.

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Introduction: Much progress has been made in the determination of the disruption of small bodies by tidal forces when they are, or pass close to, a planet. Of course, the classical and certainly best known and used theory is that for a fluid body, which was first given by Roche[1]. But small solar system bodies are not fluids, so a theory that includes the strength of solids is required. Previous contributions (Holsapple and Michel [2], [3]) presented an exact theory for the tidal approach limits for ellipsoidal solid bodies with zero cohesion, using the Drucker-Prager strength model appropriate for granular materials. There also we discuss previous approaches and references. Here we present the general case of a material that has cohesive and tensile strength also. We find that cohesive strength dominates the results for small bodies, but is not needed for the larger ones.

Strength of Rocks, Soils and Ices: A characteristic of solids with a granular structure is that the strength in shear depends strongly on the confining pressure. The shear strength at zero confining pressure is technically called the cohesion; but that (and as a consequence the tensile strength) is zero for a material such as a dry sand or loose pile of gravel or rocks. For such materials the increase in shear stress capability with increasing pressure is commonly modeled using either the Drucker-Prager model or the Mohr-Coulomb model. In either strength is characterized by an angle of friction ϕ, which can range from zero to 90°. The case with zero friction angle has no shear strength whatsoever, so it is the model for a fluid or gas. Typical dry soils have angles of friction of 30°-40°.

Since the fluid case was included in the theory as a special case, the classical results of Roche [1] was included and re-derived in their entirety; but the general solid case has much more variety and applicability. The static tidal limits for bodies using those models was presented last year (Holsapple and Michel [3] and is in Holsapple and Michel [2]).

But a solid rock or ice body would have a non-zero cohesion and tensile strength. It is the effect of that additional strength component that we include in the present theory.

Stress states at Failure: The basic approach is to calculate the stress distribution from the stress equilibrium equations with the body forces from self-gravity, spin and tidal effects, and then to look for the maximum tidal loads (closest approach) for which part or all of the body has stresses exceeding the failure criterion. We do not use nor need any relation between the stresses and strains such as liner or nonlinear elasticity, since the approach does not consider displacements nor strains.

In general, failure will occur first at a specific location, then for increasing loads a failure region will grow until it encompasses a sufficient region for global structural failure and disruption. It is that final global failure that we seek to determine.

In the theory using a model with zero cohesion, the stress states turn out to be at failure at all locations simultaneously at the same loads, clearly indicating a global failure. But that convenient property is not true of the stress state when the cohesive terms are included. For that reason, we apply a method based on the average stresses throughout the body, and determine the loads (tidal distance) for which failure occurs on average. The volume-averaged stresses are given as

$$\bar{\sigma}_x = \left[\rho\omega^2 - 2\pi\rho^2 G A_x + \frac{8\pi}{3} G\rho\rho_p \left(\frac{d}{R}\right)^{-3}\right]\frac{a^2}{5},$$

$$\bar{\sigma}_y = \left[\rho\omega^2 - 2\pi\rho^2 G A_y - \frac{4\pi}{3} G\rho\rho_p \left(\frac{d}{R}\right)^{-3}\right]\frac{b^2}{5},$$

$$\bar{\sigma}_z = \left[-2\pi\rho^2 G A_z - \frac{4\pi}{3} G\rho\rho_p \left(\frac{d}{R}\right)^{-3}\right]\frac{c^2}{5}$$

These can then be used in a failure criteria.

For zero cohesion, the averaging approach gives the exact answer because of the simultaneous failure at all locations. However, for the general case with cohesion, this averaging method gives an approximate result. Holsapple [4], [5] first presented that averaging method in calculation of spin limits for small bodies. The meaning of this averaging approach is made more clear in Holsapple [6] where it is proved that the method gives an upper bound to the global failure loads (lower bound for closest approach). Others have presented analyses using the first failure in a liner elastic analysis, which gives a lower bound. Thus, taken together, the two approaches are complimentary, giving strict bounds on failure states.

Results: The results can be presented in various ways. A basic result is for the closest distance a small (secondary) body can exist close to a large (primary) body, as a function of the various parameters of ratio α=c/a of the body semi-axes, radius r, density ρ, spin ω and strength measures of cohesion k and angle of friction θ.

An important special case is for a spin-locked satellite orbiting a planet. That minimum distance (in a scaled form) as a function of an inverse scaled cohesive strength is shown in Fig. 1. It is observed that at the left, for large cohesive strength or for small bodies, the distance can decreases all the way to the primary surface. But at the right of this plot, for small cohesion or large bodies, the distances approach a value that is independent of the cohesion but still depends on the shape and angle of friction. This is another case of strength and gravity regimes. The cohesive strength is a factor only for small bodies. For large bodies the gravitational forces dominate the cohesive strength.

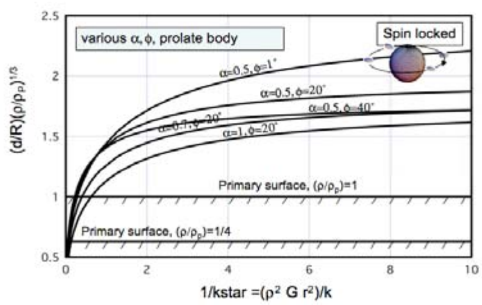

Fig. 1. Tidal disruption limits for spin-locked orbiting prolate bodies as determined by the cohesive shear strength *k*.

An alternate way to present the results is to assume a cohesive strength, and plot the distance as a function of the secondary size. The study of the observed maximum spins of the asteroids suggests a cohesive strength model that decrease with body size to the -1/2 power (Holsapple [7]). A result assuming an aspect ratio of 0.7 and using that strength model in this tidal problem is depicted in Fig. 2. Four satellites with about that aspect ratio are also shown on this plot.

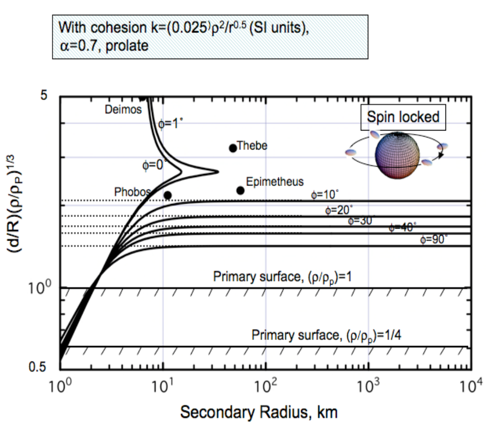

Fig. 2. Tidal disruption limits for spin-locked orbiting prolate bodies with the aspect ratio of 0.7 as a function of body size using a particular cohesive strength model.

Other results will be presented at the conference.

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