

MASS DISTRIBUTION AND PLANET FORMATION IN THE SOLAR NEBULA. S. J. Desch,
School of Earth and Space Exploration, Arizona State University, Tempe, AZ 85287. (steve.desch@asu.edu).

The surface density profile of the solar nebula is a fundamental quantity bearing on nearly all aspects of solar system formation: chemistry, transport, planet formation, etc. This largely unknown quantity has traditionally been estimated using the “minimum mass solar nebula” (MMSN) model [1,2]. In this model, the rocky component of each planet is augmented with H and He gas so that it has solar composition, and then the augmented mass is spread out over the annulus in which each planet orbits. On this basis the MMSN surface density is inferred to have been $\Sigma(r) = 1700 (r/1 \text{ AU})^{-3/2} \text{ g cm}^{-2}$ [2]. Integrating over r , the total mass between 0.1 and 30 AU is found to be $0.0123 M_{\odot}$. This is comparable to the observed masses of disks in star-forming regions [3], although of course such observations are insensitive to solids larger than a centimeter, which may hide a substantial fraction of the mass.

Although it is commonly acknowledged that the MMSN model represents a *minimum* mass, and that the solar nebula surface densities might have been greater, it is nevertheless standard to use the MMSN surface density when calculating relevant quantities: for example, the timescale to grow Uranus and Neptune [4]. Surface densities could be much higher (and planetary growth timescales much shorter) if, for example, only a fraction f_P of all solids actually wound up in planetesimals accreted by planets. Since the fraction of solids in the form of dust would be swept out of the solar system along with the gas, the surface density really should be increased over the MMSN value by a factor of f_P^{-1} . Calculations of the growth of solids from dust to planetesimal sizes are difficult and are generally lacking, but some estimate $f_P \sim 0.5$ [5], implying the MMSN is too low already by a factor of 2. The MMSN model further assumes that all the planets achieved their isolation masses, and that all the planetesimals were absorbed into planets. This assumption has not been tested, and indeed the cores of Uranus and Neptune are not predicted to achieve their isolation masses within the lifetime of the solar nebula [6]. Finally, and most importantly, the MMSN model implicitly assumes that the planets formed where they orbit today. In fact, there is strong evidence [7-11] that Saturn, Uranus and Neptune suffered considerable outward migration.

The Nice model [9-11] of solar system formation explains several attributes of solar system architec-

ture, if the giant planets formed in a more compact configuration than they are found in today, and then passed through a chaotic period some 650 Myr later as Jupiter and Saturn crossed their mutual 2:1 resonance. In particular, the semi-major axes, eccentricities and inclinations of the giant planets, the timing and flux of the Late Heavy Bombardment, and the number of trojan asteroids are all quantitatively explained. In a particularly successful model [10], Jupiter is assumed to form at 5.45 AU, Saturn at 8.18 AU, Neptune or Uranus at 11.5 AU, and Uranus or Neptune at 14.2 AU. The most successful models include a close encounter with Saturn by one of the ice giants (Uranus or Neptune), followed by a close encounter between the two ice giants. Following this close encounter, Uranus and Neptune switch positions in 50% of their simulations. Given the similarity in the ice giants’ masses, there is no practical way to determine whether this possibility actually occurred using dynamical arguments alone.

We have reconstructed the MMSN model, no longer assuming the planets formed where they orbit today. Instead we recalculate the surface density of the solar nebula, taking the starting positions of the giant planets assumed in the Nice model, as listed above. Details of our analysis can be found in the paper by Desch (2007) [12]. The final result is displayed in Figure 1, from [12]. In this figure, vertical bars run through the starting positions of each planet, and horizontal bars span the assumed feeding zone of each planet. Using the area of the feeding zone, the augmented mass of each planet is converted into a surface density; the vertical extent of each bar reflects the uncertainty in the augmented mass. We do not include the terrestrial planets in this analysis, as we do not expect them to reflect the isolation masses in those regions, but we include the 4 giant planets as well as the $\approx 35 M_{\oplus}$ planetesimal disk beyond the ice giants. These 5 data points are very well fit by a single power law,

$$\Sigma(r) = 343 (f_P/0.5)^{-1} (r/1 \text{ AU})^{-2.17} \text{ g cm}^{-2}.$$

If one were to (incorrectly) treat the uncertainties in the augmented masses as standard deviations, one would infer an exceptionally low $\chi^2_{\nu} = 0.03!$ This fit assumes that Uranus and Neptune did indeed switch places during the solar system’s chaotic period. If Uranus and Neptune did not switch places,

then there is no monotonic function $\Sigma(r)$ that can fit the planetary data. (The equivalent χ^2_ν would be increased to > 2 .) We view this as strong, if circumstantial, evidence that **Neptune formed closer to the Sun than Uranus, and they switched places during a close encounter 3.9 Gyr ago.**

Additional constraints also conform to this fit. If Uranus formed at the geometric mean of the inner and outer radii of its feeding zone, the location at surface density at the inner edge of the planetesimal disk are consistent with the timing of the late heavy bombardment / 2:1 resonance crossing as inferred in the Nice model. Based on an examination of published results, [12] argues that Neptune's migration transitions from runaway migration to damped migration where the surface density of planetesimals drops below 0.22 g cm^{-2} ; our fit predicts this surface density at 30 AU, exactly where Neptune is found today. Finally, the surface density of gas at 2-3 AU has been estimated by [13] on the basis of models in which chondrules are melted by solar nebula shocks. **A single power law very well fits 8 separate constraints on the surface density.**

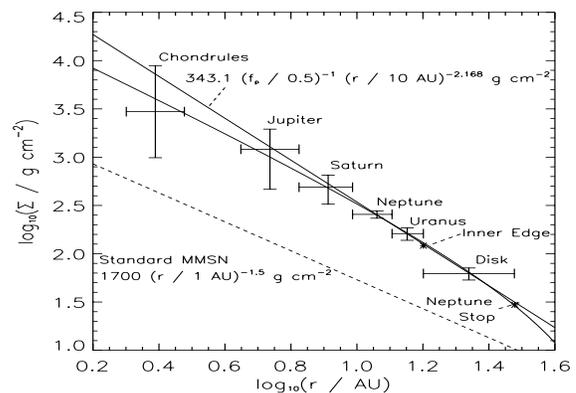
This result is to be compared to the standard MMSN [1], in which the fit conforms poorly to a monotonic, power-law distribution. Our inferred surface density is also much denser at all radii, and falls off much more steeply with distance from the Sun. In fact, our inferred surface density, like that of the standard MMSN, is inconsistent with a steady-state α accretion disk. In a disk with $\Sigma \propto r^{-p}$, a steady-state disk requires $p \approx 1$ [14]. In fact, it is possible to show that if the surface density we infer is in steady state, **mass must have been moving OUTWARD through the giant planet region at the time the solids grew to planetesimal size [12].** This suggests that the behavior of the disk is driven by external photoevaporation, which removes mass from an outer edge of the disk (at $r_d \approx 30 - 100 \text{ AU}$), at a rate $-\dot{M}$. Making these assumptions, the steady-state profile of the disk is shown to be (for $r > 2 \text{ AU}$)

$$\Sigma(r) = \frac{-\dot{M}}{3\pi\nu(r)} \left[\left(\frac{r_d}{r} \right)^{1/2} - 1 \right],$$

where $\nu(r) = \alpha c_s^2 / \Omega$ is the viscosity of the disk, Ω is the orbital frequency, and the sound speed c_s is computed from the temperature profile of [15]. This surface density has two free parameters, $-\dot{M}/\alpha$ and r_d . Assuming $-\dot{M} = 5 \times 10^{-9} M_\odot \text{ yr}^{-1}$, $\alpha = 4 \times 10^{-4}$, and $r_d = 61 \text{ AU}$, we compute the surface density depicted in Figure 1 as a solid curve. This

distribution fits the data at least as well or better than a single power law. **The surface density of the solar nebula is completely consistent with a disk that is being externally photoevaporated, implying the solar system formed in a cluster with massive stars [12].**

Finally, planet formation is much more rapid than in the MMSN model, as a result of the planets forming closer to the Sun where the orbital frequencies are greater, the surface density of solids is larger, and where the surface density of gas is higher, more effectively damping the eccentricities of the planetesimals, enabling accretion. Using standard formula for the growth of planetary cores [16], we find that **all 4 giant planet cores can form within the lifetime of the disk.** According to [12], core masses and formations times were: Jupiter, $26 M_\oplus$, 0.5 Myr; Saturn, $23 M_\oplus$, 2 Myr; Neptune, $15 M_\oplus$, 5 Myr; and Uranus, $13 M_\oplus$, 10 Myr. **The assumptions of the MMSN are validated, and the distribution of mass $\Sigma(r)$ is seen to reveal much about the dynamics and birthplace of the solar nebula.**



References: [1] Weidenschilling SJ 1977 Ap&SS 51, 153. [2] Hayashi, C 1981 PThPhS 70, 35. [3] Eisner JA & Carpenter JM 2006 ApJ 641, 1162. [4] Goldreich P et al. 2004 [5] Weidenschilling SJ 2000 SSR, 92, 295. [6] Lissauer JJ & Stewart GR 1993 PP III (UofA), 1061. [7] Fernandez JA & Ip WH 1984 Icarus 58, 109. [8] Malhotra R 1993 Nature, 365, 819. [9] Tsiganis K, Gomes R, Morbidelli A & Levison HF 2005 Nature 435, 459. [10] Gomes R, Levison HF, Tsiganis K & Morbidelli A 2005, Nature, 435, 466. [11] Morbidelli A, Levison HF, Tsiganis K & Gomes R 2005, Nature 435, 462. [12] Desch SJ 2007 ApJ 671, 878. [13] Desch SJ & Connolly HC 2002 MAPS 37, 183. [14] Lynden-Bell D & Pringle JE 1974 MNRAS 168, 603. [15] Chiang EI & Goldreich P 1997 ApJ 490, 368. [16] Kokubo E & Ida S 2002 ApJ 581, 666.