CENTRAL CONFIGURATIONS OF N BODIES AS MODELS OF SECONDARY COORBITAL PLANETS AND PLANETARY RINGS.

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Introduction: In the last decades several systems of quasi-co orbital satellites of the giant planets have been discovered. (Jupiter: Metis, Adrastea and Lysithea, Elara; Saturn: Telesto, Tethys, Calypso and Dione, Helene, Polydeuces; Uranus: Cupid, Belinda). Moreover, near of the all giant planets the systems of rings have been revealed [1]. The discoveries of the new exosolar systems cause the problem for searching of planets of terrestrial type. The simple analytical models of motion of these celestial mechanical systems are not found.

In 2007 several interesting papers devoted the central configurations have been appeared [2], [3]. The authors of these papers found new central configurations without, as a rule, paying attention: a) stability of the central configurations; b) determination of the positions of the corresponding librations points; c) applications of these dynamical models for the real celestial mechanical systems.

In our work [4] 4-body central configurations are investigated. Here we consider N-body central configurations (N>4) as models of co orbital satellites. In according with Winter [5] *I* vectors $\boldsymbol{R}_J$ determined the positions of *I* bodies of mass $m_1,\ldots, m_I$ at the baricentric coordinates system form the central configuration in respect of positive constants of $m_1,\ldots, m_I$, if the attractions force, acts upon *J*-body at the fixed time moment, is in proportion to the mass m_J and vector $\boldsymbol{R}_J$.

$$\boldsymbol{F}_J=\sigma' m_J \boldsymbol{R}_J,\ J=1, I. \tag{1}$$

Scalar σ' is not depending upon *J*. The uniqueness of σ' value results from (1) and Newton's laws.

A Stable Four Body Central Configuration: For regular triangle with side equals *a* and in vertexes of which arbitrary gravitating mass of m_1 and $m_2=m_3 \neq m_1$ are placed there is a stable point of librations (with negligible mass). This point is stable if mass ratio of $m_1/m_2=m_1/m_3>367.0540108$. This point lies in the straight line "body with mass m_1 and the middle of the distance between the bodies with mass m_2 and m_3. In the system of the center of mass of this configuration the distances of the bodies (in units of *a*) are determined by the following values: $R_c/a=0.996833$ (for point of librations), $R_2/a=R_3/a=0.995938$ and $R_1/a=0.00469321$. In the considered model three bodies (mass m_2, m_3 and the point of librations) are moving along almost one and the same circular orbit. Moreover, this dynamical system is completely stable (!). It should be noted in pair two body problem the life time (τ) of such systems is restricted approximately by a value of $(m_1/m_2)P$, where P is orbital period of body with mass m_2 (or m_3). For the co orbital satellites of the giant planets $\tau\approx10^4$ years [4].

This stable four body configurations is interesting to use for describing of the motion of an arc of a ring in the gravitational field of two "shepherd" satellites.

A Five Body Central Configuration: Considering a pentagonal central configuration with arbitrary mass m_1, m_2, m_3, m_4, m_5 placed in the vertex of the pentagon for which $m_2/m_1=m_5/m_1$, $m_3/m_1=m_4/m_1$ it has been cleared out that the central configurations exist for the ratio $m_2/m_1>0.00402$. In this extremely case $m_2/m_1=m_5/m_1=0.00402$, $m_3/m_1=m_4/m_1=0.00001057$ and distances between the bodies with mass m_1 and m_j (r_{1j}/r_{12}) are equal $r_{15}/r_{12}=1$, $r_{13}/r_{12}=r_{14}/r_{12}=1.001976132$. Using the theorems of Lyapunov we draw a conclusion motion in the direction perpendicular at the plane of the pentagon is stable. So, the satellites of the planets with mass differed by order of 10^3 may move along one and the same circle orbit in the considered model.

A Six Body Central Configuration: A hexagonal central configuration with arbitrary mass m_1, ..., m_6 placed in the vertex of the hexagon for which $m_2=m_3=m_5=m_6$; $m_1=m_4$ exists for the ratio $m_1/m_2>3.3297$. (Straight lines m_2m_3 and m_5m_6 are parallel, as well as sides of this polygon m_2m_6 and m_3m_5). The distance between points with mass m_1 and m_2 we denote by letter a, and the distance between points with mass m_2 and m_3 equals *x*. An angle between segments m_1m_2 and m_1m_6 is equals to *α*. For the case $m_1/m_2=m_1/m_3=m_1/m_5=m_1/m_6=100000$, we have $x/a=0.023854$, $\alpha=121.184905°$ and the positions of the bodies with mass m_i (*in units of a*) in respect of the centre mass of the systems are equal to $R_1/a=R_4/a=0.502945$, $R_2/a=R_3/a=R_5/a=R_6/a=0.504611$. So, this model is more suitable for searching of planets Earth type in the double star exosolar planetary systems moving along one and the same circle orbit.

A Seven Body Central Configuration: For a heptagonal central configuration with arbitrary mass m_1, ..., m_7 placed in the vertex of the heptagon for which $m_2/m_1=m_7/m_1$, $m_3/m_1=m_6/m_1$, $m_4/m_1=m_5/m_1$ it has been cleared out that the central configurations exist for the ratio $0<m_2/m_1<1$. For the case $m_2/m_1=m_7/m_1=0.00250$, $m_3/m_1=m_6/m_1=0.00741308$, $m_4/m_1=m_5/m_1=0.1778958$

and the distances between the bodies with mass m_1 and m_j (*in units of* r_{12}) are equal r_{17}/r_{12}=1, $r_{13}/r_{12} = r_{16}/r_{12}$ =1.1552636, $r_{14}/r_{12} = r_{15}/r_{12}$ =1.1699528. So, the satellites of the planets with mass differed by order of 10^6 may move along one and the same circle orbit in this model.

5N+1 Body Central Configuration: Now we pay attention the known central configuration which is formed by equal mass $m=m_i$ placed in the vertexes of a regular N- polygon and in the center of the mass of this dynamical system a body with mass M is placed. We denote the radius of the orbit of the bodies with mass m by letter R. G is the gravitational constant. Then angular velocity of the system is ω.

$$\omega^2=Gm/R^3(1/4\sum(1/sin(\pi l/N))+M/m),\ l=1,...,N-1. \quad (2)$$

It is easy to show there are $5N$ points of librations (bodies of negligible mass) in this system. Let R_c is a radius of a circle orbit of a librations point. For the determination of the positions R_c of N *stable* points of librations the equation (3) should be solved in respect of R_c

$$M/m(1-1/(R_c/R)^3)+1/4\sum(1/sin(\pi l/N))-1/(R_c/R)\cdot((R_c/R-cos(\pi(2l-1)/N))/(1-2R_c/Rcos(\pi(2l-1)/N)+(R_c/R)^2)^{3/2})=0,\ l=1,\dots,N. \quad (3)$$

With help of Lyapunov's methods the stability of these points of librations is investigated. (See Table 1).

Table 1. The positions (R_c/R) of *stable* equilibrium of N small bodies depending on the minimal value of the ratio M/m. (The number of major bodies are also N). These "external" points placed outside the gravitating regular polygon and they lie on the straight line "body with mass M and the middles of the sides of the N-polygon".

N	3	4	5	6
$(M/m)_{min}$	43.1810	105.816	210.1070	366.514
R_c/R	1.00520	1.00282	1.001774	1.00122
N	7	10	100	1000
$(M/m)_{min}$	585.495	1722.37	$1.73*10^6$	$1.74\cdot10^9$
R_c/R	1.00089	1.00043	1.000004	$1+4\cdot10^{-8}$

For the given N body central configuration the minimal value of mass ratio $(M/m)_{min}$ for which N points of librations are stable equals approximately

$$(M/m)_{min}\approx3^{1/2}\cdot N^3 \quad (4)$$

This condition is only necessary, but is not sufficient, in consequence of existence of zero of real parts of complex roots of characteristic Lyapunov's equations. Based on the Table 1 and formula (4) we may draw a conclusion: the lesser mass of the ring particle the greater the number N of these particles (the greater the ratio $(M/m)_{min}$) required for the stability of the librations points in the Newtonian's gravitational fields created by mass M and $m_i=m$, i=1,…, N (here we do not consider the stability of the system of the major bodies with mass M and $m=m_i$, i=1,…, N).

For estimating the number N of the major bodies radius of R_p=1 m and the same density as the density of the planet, we assume these bodies are moving around the planet along the circle orbit of radius $r=2R_{pl}$, where R_{pl} =60000 km is a radius of the planet (the parameters of this model system approximately coincide with the parameters of the system "Saturn and its ring"). Using the estimating formula (4) we find the system of *librations* points is stable for $N\approx5\cdot10^7$. (The length of the corresponding circumference equals $7.5\cdot10^8$ m $<2R_pN$; mass of the ring is equal to $2\cdot10^{11}$ kg $<<M_{pl}$, - the density of the planet ρ=1000 kg/m^3).

Conclusion: The stable N body central configurations exist and they may be useful as models for describing of motion of the planetary co orbital satellites and the particles of the planetary rings (N is odd number). For the even N the considered central configurations may be used for searching of undiscovered planets in double star systems.

In the next paper the applications of the space central configurations for modeling of motion of real celestial mechanical system will be presented.

References: [1] http://nssdc.gsfc.ov/ planetary/factsheet/ Saturnian Satfact.html. [2] Mercedes Aribas et. al. (2007) *CM, 99,* 1-12. [3] Marshal H. and Manuele S. (2007) *CM, 99,* 293-305. [4] Perov N. I. and Medvedev Yu. D. (2007) *LPS XXXVIII,* Abstract #1026. [5] Winter A. (1941) *The Analytical Foundations of Celestial Mechanics*. Prinston. New York.