Cassini's motions and resonant librations of synchronous satellites in Solar system. Yu.V.Barkin, Sternberg Astronomical Institute, Moscow, Russia/ yuri.barkin@ua.es/Fax:+07-095-9328841

Introduction. In the paper the rotations of synchronous satellites of the Jupiter, Saturn, Uran and Neptune are studied. On the base theory of resonant rotation of the rigid satellite on precessing elliptical orbit (Barkin, 1978, 1979) parameters of Cassini's motions and periods of free resonant librations have been determined for big grope of satellites of planets. Here I use observed values of coefficients of second harmonics of gravitational potensials (J_2 and C_{22}) and of dimension less moment of inertia $I = C/\left(mr^2\right)$ of Io, Europa, Ganimede, Callisto and also Rhea and Titan, obtained on the base of data of space missions to these bodies (Anderson et al., 2001). Here C is the polar moment of inertia, m and r is the mass and the mean radius of satellite. Mentioned parameters J_2, C_{22} and I also have been evaluated for a wide set of another's satellites of big planets for their models as homogeneous ellipsoids of known forms and sizes (www.nasa.gov). These models also have been obtained here effective applications. For corresponding models the notation (**e**) is used here. For another from considered satellites (without indexes) we use also ellipsoidal models of hydrostatic equilibrium state of synchronous satellite on the Goldreich, Peale results (1968). The full list of discussed parameters for satellites of planets is presented in the paper (Barkin, 2004). Perturbed orbital motions of considered satellites we discribe by mean orbital elements reffered to local Laplacian planes of corresponding satellites (http://ssd.jpl.nasa.gov/sat_elem. html). From them: the eccentricity (e), the inclination of orbit plane (i), the mean orbital motion and its period (n and T_n), the angular velocity and period of preseccion of orbit plane of satellite on local Laplacian plane (n_Ω and T_Ω). In our approach all mentioned parameters are considered as constants and more fine effects in orbital motions of satellites do not take into account in this paper. The purpose of paper is to study syncronous motions of satellites in Solar system and for each of them to determine the values of the basic Cassini's parameter ρ_0 (it is the average angle of inclination of the axis of rotation relatively to normal of the precessing orbit plane) and the periods of resonant librations in the longitude (T_g), in the pole wobble (T_l) and period of space precession (T_h) (and their errors). Here we use the analytical formulas for mentioned parameters which were developed by study of the Moon Cassini's motion in my early papers (Barkin, 1978, 1979). Specially for the case of small eccentricities and inclinations of orbits of synchronous satellites we have obtained the simple reduced formulas for all four considered parameters. The small value of Cassini's angle ρ_0 is determined by grope of formulae:

$$\rho_0 = -\sin i / \left(\cos i + J\right), \qquad (1)$$

$$J = f_\Omega \left[\left(J_2 / I\right) + 2\left(C_{22} / I\right) \right], \qquad f_\Omega = 3n/(2n_\Omega).$$

Periods of resonant librations:

$$T_g = T_n \Big/ \left[2\left(3C_{22}/I\right)^{1/2} \right], \quad T_h \, ; \, T_\Omega \left(\sin \rho_0 / \sin i\right),$$

$$T_l = I T_n \Big/ \left[2\left(J_2^2 - 4C_{22}^2\right)^{1/2} \right]. \qquad (2)$$

Here we must remark that for all considered satellites in this paper in acordance with generalized Cassini's laws ascending node of the orbit coinside with ascending node of the equator on the local Laplacian plane. Similar situation has place in the Mercury resonant motion.

Synchronous satellites of Jupiter. The first determinations of the resonant periods of librations in vicinity of synchronous motions for some satellites of the Jupiter and Saturn (for rigid ellipsoidal models) have been undertaken by the author in 1981 (Barkin, 1981). On the basis of the specified parameters of Galilean satellites in the given paper the Cassini's angle of inclination (with error) $\rho_0 \pm \Delta\rho_0$ and values of periods of their resonant librations (with errors) $T_g \pm \Delta T_g$, $T_l \pm \Delta T_l$ and $T_h \pm \Delta T_h$ have been more precisely determined (Table 1). Below (in all tables) the values of the inclinations ρ_0 of satellites are given in arc seconds and values of resonant periods in days (d) or in years (yr).

Table 1. Cassini's inclination and periods of resonant librations of Galilean satellites of Jupiter.

Satellite	$\rho_0 \pm \Delta\rho_0$	$T_l \pm \Delta T_l$	$T_g \pm \Delta T_g$	$T_h \pm \Delta T_h$
Io	**7"607 ± 0"019**	**225.7 ± 0.6 d**	**13.32 ± 0.02 d**	**159.3 ± 0.4 d**
Europe	**201"3 ± 7"5**	**4.86 ± 0.13 yr**	**52.66 ± 0.88 d**	**3.60 ± 0.13 yr**
Ganymede	**109"8 ± 5"2**	**30.1 ± 1.17 yr**	**186.4 ± 2.97 d**	**23.6 ± 1.1 yr**
Callisto	**1393" ± 163"**	**317 ± 11 yr**	**2.46 ± 0.051 yr**	**625 ± 73 yr**

The obtained estimations of inclinations and the periods of resonant (free) librations will well be coordinated to the similar estimations obtained in recent works (Motomoto et al., 2002; Henrard et al., 2005; Noyelles et al., 2007, Noyelles, 2007). For example in the Table 2 the comparison of different determinations of parameters of Io rotation are presented. The first estimations of the periods of resonant librations of the satellites: Io, Japetus, Phobos, Deimos etc. (modelled by homogeneous ellipsoids), have been obtained by the author in 1981. In particular the period of Io librations in longitude has been determined in **11.3** days, and the period of its pole wobble in **163** days (Barkin, 1981).

Table 2. Determinations of Io rotation parameters.

$i+\rho_0$	T_g (d)	T_l (d)	T_h (d)	Authors (yr)
---	11.3	167	---	Barkin (1981)
---	13	219		Musotto et al. (2002)
157”	13.25	229.9	159.4	Henrard (2005)
---	13.31	228.5	160.2	Noyelles et al., (2007)
140”	13.31	225.5	159.2	Noyelles (2007)
137”21 $\pm$ 0”02	13.32 $\pm$ 0.06	225.7 $\pm$ 0.6	159.3 $\pm$ 0.4	Barkin (2007)

Table 3. Rotation parameters of Saturn satellites.

Satellites	ρ_0	T_l	T_g	T_h
Mimas (e)	117”	10.7 d	2.16 d	7.48 d
Enceladus (e)	0”81	27.5 d	4.21 d	19.4 d
Tephys (e)	23”7	105 d	9.24 d	71.3 d
Diona	0”38	323 d	19.8 d	227 d
Rhea (C)	85”5	3.59 yr	51.9 d	2.60 yr
Titan	261”	221 yr	2.08 yr	155 yr
Titan (C)	296”	307 yr	2.09 yr	176 yr
Phoebe (e)	279”	12.4 yr	4.11 yr	11.7 yr
Janus (e)	8”50	1.95 d	3.08 d	2.52 d
Epimetheus (e)	15”0	8.11 d	0.85 d	2.19 d

In the work the estimations for Cassini’s angle and for periods of resonant librations also have been obtained for Jupiter satellites Adrastea and Amalthea, but already on the basis of their models as homogeneous ellipsoids with certain semi-axes (http://www.nasa.gov). For example, the value of the angle ρ_0 for Adrastea has made **1” 68**, and for the fifth satellite of Jupiter Amalthea **3”54**. The period of librations in longitude for Amalthea by the executed estimations makes **0.312 days**, the period of the pole wobble makes **1.070** days, and the precession period is equal **0.259** days. The period of librations in longitude for Adrastea by the executed estimations makes **0.287** days, accordingly, the period of the pole wobble of this satellite makes **0.464** days, and the period of spatial precession is equal **0.258** days. Periods of librations of Amalthea in longitude, in the pole wobble and in precession have been evaluated earlier as **0.44** days, **1.88** days and **0.315** days (Barkin, 1981).

Synchronous satellites of Saturn. Some from considered satellites of Saturn were modeled by homogeneous ellipsoids. These models mentioned in the Table 3 by index (e) are based on known data about forms and sizes of these satellites (www.nasa.gov). Models of Titan (C) and Rhea (C) were constructed on the base of the modern data about their gravitational fields obtained in result of Cassini mission. For others from considered models have been used ellipsoidal models of hydrostatic equilibrium state of synchronous satellite on the Goldreich, Peale results (1968). Calculated values of rotation parameters for set of Saturn satellites are presented in the Table 3. Values of the resonant periods of model the Titan (e) will well be coordinated to similar characteristics from the work executed by Noyelles et al. (2007): 2.098 yr, 306.4 yr, 167.5 yr. The first determinations of periods of librations in a longitude and in the pole wobble of the Titan have been determined in the author work: 2.21 yr and 252 yr (Barkin, 2004).

Synchronous satellites of Uran and Neptune. Rotation parameters of some synchronous satellites of Uran and Neptun are presented. These values were obtained mainly for hydrostatic ellipsoidal model. In parallel also were considered close ellipsoidal models but constructed on the base of data about forms and sizes of satelites Миранда (e), Ariel (e) and Proteus (e).

Table 4. Parameters of Uran and Neptune satellites.

Satellite	ρ_0	T_l	T_g	T_h
Miranda	83”6	51.8 d	5.70 d	34.7 d
Miranda(e)	72”3	53.9 d	5.05 d	30.0 d
Ariel	1”40	296 d	18.2 d	199 d
Ariel (e)	2”05	2.40 yr	19.6 d	290 d
Umbriel	8”83	3.59 yr	49.0 d	2.43 yr
Oberon	177”	124 yr	1.42 yr	142 yr
Titania	36”3	744 yr	13.3 yr	560 yr
Triton	495”4	6.29 yr	77.2 d	4.20 yr
Proteus (e)	0”184	10.8 d	2.93 d	9.18 d

Conclusions. The values of parameters of rotation of synchronous satellites of big planets have been obtained for their simple models. The consideration of real properties of shells of these celestial bodies should result in the certain corrective amendments of the obtained results. The new data on gravitational fields and an internal structure of satellites of major planets will be necessary for this purpose in the future. The fulfilled studies on rotation of synchronous satellites partially have been supported by Russian-Japanese grant N 07-02-91212.

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