ROTATIONAL DYNAMICS OF A COMET NUCLEUS SUBJECT TO OUTGASSING JETS. S. M. Byram and D. J. Scheeres, *University of Michigan, Ann Arbor, Michigan (byramsm@umich.edu*)

Results: A theory for the long-term evolution of comet nucleus rotation states is presented and verified. The theory can be used to develop realistic simulations over long time spans, and can incorporate slowly changing jet geometry and nucleus relaxations. The theory assumes an axisymmetric comet body. When axisymmetric comet nuclei are considered, we find ~50% have an end state that spins up and ~50% have an end state that spins down. We also find that over 50% naturally tend to a relaxed state without dissipation if there are multiple jets on the surface. A nucleus with a single jet is more likely to evolve to an excited state.

Background: Changes in the rotational state of a comet have been studied previously. Changes in the spin period and the angular momentum vector have been found using nongravitational accelerations modelled as single and multiple discrete jets acting on the surface of ellipsoids or as large active patches acting on irregularly shaped bodies [1,2,3,4]. Our focus is to study the long term effects of multiple discrete outgassing jets through multiple perihelion passages by using a method of averaging. Specifically the equations of motion developed by Neishdadt, et. al.[2] will be explored and generalized. The generalized averaged equations uncover the drift in nutation angle, cone angle, and angular momentum over long periods of time without the computationally intense integration of the comet's full equations of motion.

Model: It is assumed that the comet is located significantly far from any other major celestial body such that it follows the two-body orbital equations for motion about the Sun and it follows the Euler equations for attitude with an axis-symmetric inertia distribution. The applied moments on the body are reactions to the discrete jets located at arbitrary positions on the surface. Each jet is assumed to be fixed on the comet surface with circular cross sectional active regions. It is assumed that the gas departs at a constant velocity in a direction defined by the jet's orientation, which can be any arbitrary direction away from the surface of the comet. This outgassing model allows for the prediction of the evolution of the comet's rotational state. The strength of the jet is a function of the angle it makes with the sun at time of ejection, a thermal insulation variable denoted α, the comet's heliocentric distance, and the size of the jet and its outgassing velocity. The pressure from the jets produces a reaction moment on the comet body, changing its rotational state.

Rotational Dynamics: Coupling the model with efficient integration of the equations of motion, the rotation state evolution of the comet can be modeled over long time spans. By averaging the equations of motion, insight can be gained into long term changes in the comet's rotation state without needing to numerically integrate. The equations are first averaged over the nonperturbed nucleus motion (spin angle and then precession angle). The second averaging of the equations of motion is over the comet's heliocentric motion (mean anomaly). This averaging exposes the secular drift in the nutation angle, cone angle, and angular momentum magnitude of the comet. The equations used in the averaging assume $0 \leq \alpha \leq 1$, which lead to a more generalized set than in Neishdadt, et. al.[2] where $\alpha \leq 1/2$ is assumed. To make a true comparison, the initial conditions of the averaged and the numerical cases are chosen to be consistent with each other by adding a small offset for the magnitude of the mean periodic part of the solution to the averaged equations.

Outcome of Averaged Equations: The rotation state evolution is dependent on the surface geometry, number of jets located on the comet's surface, inertia, and heliocentric orbit. These parameters can be collapsed into a single parameter, κ, that controls the evolution of the nucleus. Note that an observed change in rotation state of a comet can give insight into its surface activity and jet distribution through this κ variable.

There exist three important κ values which define the end state of the nucleus: $\kappa_1 < \kappa_2 < \kappa_3$. For every κ value, there exist at least four equilibrium points of the system with one stable, one unstable and the rest saddle points. When new equilibrium points exist (as κ decreases in magnitude) new stable and unstable equilibrium points arise while the original stable and unstable points become saddle points. If the jet geometry does not change over time, then the system will asymptotically approach the stable point given enough time.

The flow of the system can be predicted as seen in Figures 1-3 for $\kappa>0$ where three general cases occur. For the relaxed case, shown in Figure 1 with $\kappa>\kappa_1$, the angular momentum vector asymptotically approaches alignment along the axis of symmetry of the body and points in the perihelion direction. For the semi-excited case, shown in Figure 2 with $\kappa_1>\kappa>\kappa_2$, the angular momentum vector asymptotically approaches alignment with a nutation angle $<\pi/2$ from the maximum moment of inertia while still aligned along the perihelion direction. For the fully-excited case, $\kappa_2>\kappa>0$, shown in Figure 3, the angular momentum vector asymptotically approaches alignment with a nutation angle <$\pi/2$ from the maximum moment of inertia with a cone angle <$\pi/2$ from the perihelion direction. It is important to note that for $\kappa>0$, the system has an increasing angular momentum magnitude as it approaches the stable configuration. For an oblate body, $|\kappa|>\kappa_1$ is the "true" relaxed state meaning a stable evolution direction. For a prolate body, $|\kappa|>\kappa_1$ is actually a maximally excited state meaning an unstable evolution direction.

For a negative value of κ, the stable configuration changes depending on the parameter causing the negativity. The angular momentum magnitude is decreasing as the system reaches its stable configuration for $\kappa<0$. In general, the dynamics are similar for $\kappa<0$.

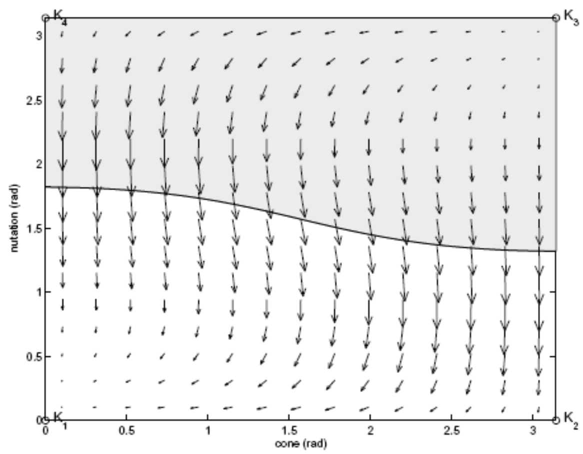

Figure 1: Nutation and Cone Angle Flow Field for Relaxed Case (κ > κ_1). Shaded = Angular Momentum Decreasing, Nonshaded = Angular Momentum Increasing.

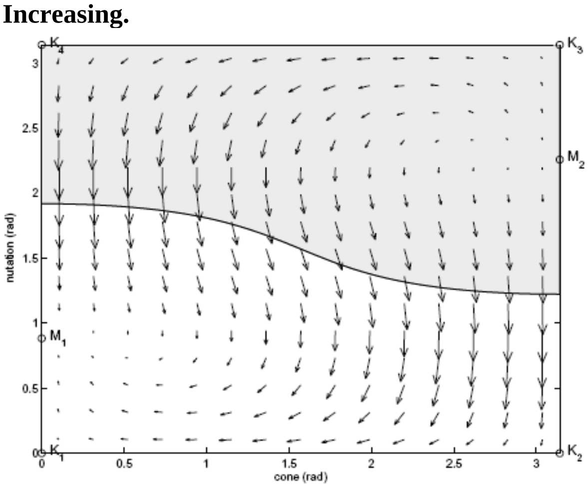

Figure 2: Nutation and Cone Angle Flow Field for Semi-Excited Case (κ_1 > κ > κ_2). Shaded = Angular Momentum Decreasing, Nonshaded = Angular Momentum Increasing.

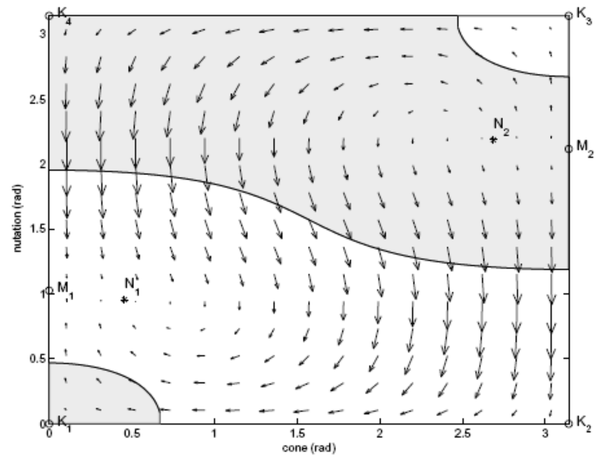

Figure 3: Nutation and Cone Angle Flow Field for Fully-Excited Case (κ_2 > κ > 0). Shaded = Angular Momentum Decreasing, Nonshaded = Angular Momentum Increasing.

For a given orbit, Table 1 shows the κ value distribution for randomly distributed jets on an oblate body. It is important to note that for a single jet on the surface, $|\kappa|<\kappa3$ is not possible due to the geometry. As a result, the single jet case is inconsistent with the other distributions for multiple jets. This distribution is only a representative example, as κ depends on the comet orbit and the nucleus shape, the possibility of other distributions exist.

The probality for $\kappa>0$ is the same as for $\kappa<0$. Therefore, there is equal probability for a come to spin up as there is for spin down. A spin reversed comet will spin up.

Table 1: $|\kappa|$Distribution

# jets	Relaxed	Semi-Excited	Fully-Excited
1	~37%	~47%	~15%
20	~54%	~16%	~30%
50	~53%	~16%	~31%
100	~56%	~15%	~29%

References: [1] Chesley S. R. and Yeomans D. K. (2005) *Dynamics of Populations of Planetary Systems Proceedings IAU Colloquium, 197*, 289-302. [2] Neishtadt A. I., Scheeres D. J., Sidorenko V. V., and Vasiliev A. A. (2002) *ICARUS*, *157*, 205-218. [3] Drahus, M. and Waniak, W. (2006) *ICARUS*, *185*, 544-557. [4] Gutierrez P. J., Ortiz J. L., Rodrigo R., Lopez-Moreno J. J., and Jorda L. (2002) *Earth, Moon and Planets*, *90*, 239-247.