DETERMINING SIMPLE IMPACT CRATER SHAPES FROM SHADOWS. J. E. Chappelow[1,2]
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Introduction: Chappelow and Sharpton [1] developed a method for determining the cross-sectional shapes of certain simple impact craters from the forms of the shadows cast within them. These included parabolic, cone-shaped and flat-floored craters. Here this method is generalized to include any type of conic-section-shaped crater, of which parabolic and conical are just special cases.

The Math: If the imaginary crater has the shape of a conic section of revolution about the vertical (z) axis (Figure 1), the origin of coordinates is placed at its bottom, and the x-axis points in the sunward direction, then the algebraic equation describing the interior crater surface is:

$$\frac{x^2+y^2}{a^2}+\frac{(z-c)^2}{c^2}=1 \qquad (1).$$

Here the parameters a and c fix the shape of the crater, and are to be related to the shape of the shadow front that cuts the crater. Thus the goal is to find an equation that describes the shape of this shadow front (as viewed from the zenith) in terms of a, c and other parameters.

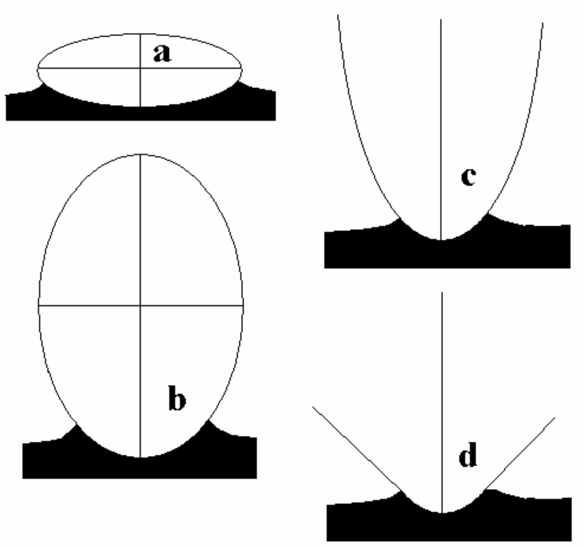

Figure 1: Several examples of conic section craters: (a) horizontal elliptical, (b) vertical elliptical, (c) parabolic, (d) hyperbolic. Taken to extreme cases, (a) would resemble a flat-floored crater, while (d) would converge to a cone-shape. These, and the parabolic case (c), were treated by [1] as special cases.

After much algebra, it can be shown that the equation describing the shadow front is that of an ellipse centered on the x-axis, with semi-x-dimension α given by:

$$\alpha=\left(\frac{a^2-c^2\tan^2\theta}{a^2+c^2\tan^2\theta}\right)R \qquad (2),$$

and semi-y-dimension $\beta = R$, where R is the crater's radius and θ is the sun-zenith angle. The ellipse is centered at $x = x_c$, $y = 0$, where:

$$x_c=\frac{2a^2(d-c)\tan\theta}{a^2+c^2\tan^2\theta} \qquad (3),$$

and where d is the crater depth. The shadow fills the intersection of this ellipse and the sunward part of the crater rim (Figure 2). The dimensions given above are illustrated on Figure 2(d).

Usefulness: Since what we are really after here are the crater shape parameters a, c, and d (R and θ are generally known), it would be useful to invert Eqns. (2) and (3). This would allow us to determine a, c, and d from measurements of the shadow parameters, α and x_c. However, this is not possible without some other known relationship among the three unknowns.

Fortunately, evaluation of Eqn. (1) at the crater rim yields such a relationship. On the crater rim, $x^2 + y^2 = R^2$ and $z = d$, and (1) becomes:

$$\frac{R^2}{a^2}+\frac{(d-c)^2}{c^2}=1 \qquad (4).$$

In principle it is possible to solve this set of equations for the crater shape parameters, given measurements of the shadow shape parameters.

Conclusions: The method derived above extends and generalizes the work of [1]. Given proper imagery (i.e. low enough sun angle, vertical look-angle; see Figure 3), it offers the possibility of calculating morphometries of many impact craters from single images of planetary surfaces (for example, images taken during fly-bys of Mercury). In particular, crater depths may be calculated from shadows cast within them, even if the shadow does not cross the center (bottom) of the crater. Implementation of this procedure is under way.

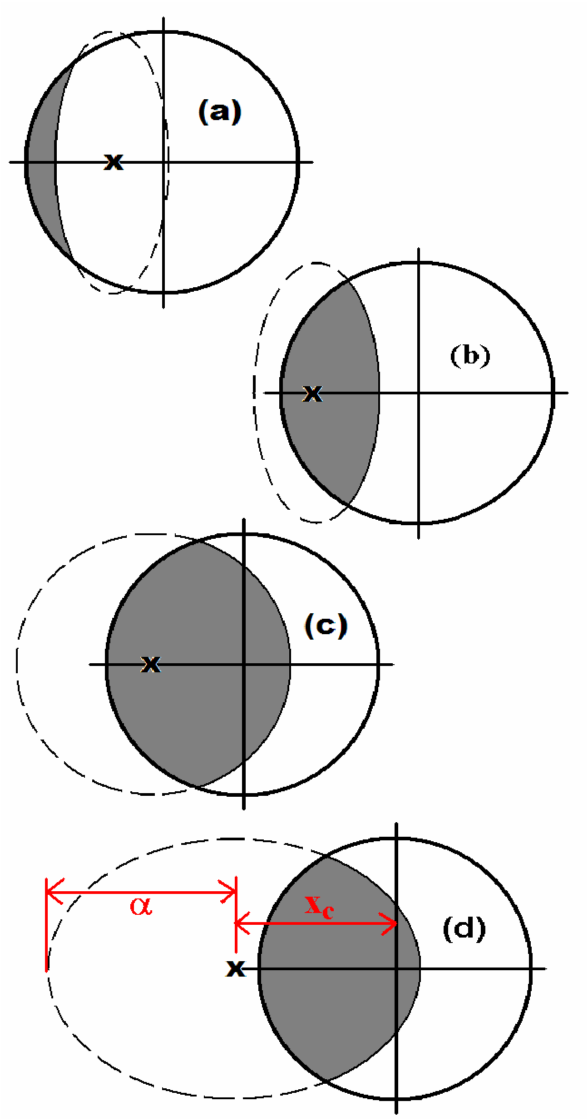

Figure 2: A view from the zenith of typical shadow shapes corresponding to the crater shapes shown in Fig.2. These shadow shapes depend on crater depth and radius, and solar zenith angle, as well as *a* and *c*, and are therefore simply examples.

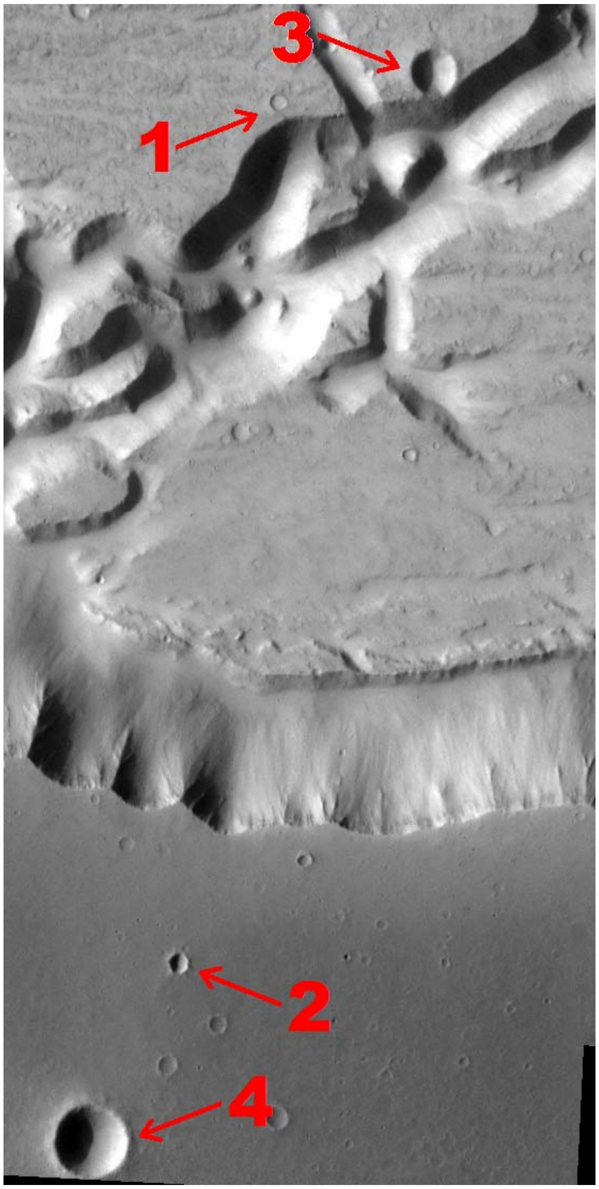

Figure 3: Several of the shadow forms (and therefore craterforms) discussed in this abstract occur in this single THEMIS image. Crater 1 is nearly flat-floored, similar to (a) in Fig. 2, appears to be nearly totally infilled, and is very shallow. Crater 2 is the next gradation, with a shadow front that is nearly a straight line, it is similar to Fig. 2(b). Crater 3 also has a rather straight shadow front, while the near-circular shadow front in crater 4 indicates that it is nearly parabolic in section.

References: [1] Chappelow, J.E. and Sharpton V.L. (2002) *Met. & Planet. Sci. 37,* 479-486.