A POSSIBLE ORIGIN FOR THE –3/2 POWER LAW DISTRIBUTION OF SOLIDS IN THE SOLAR NEBULA. Takashi Fukui, Hidekazu Tanaka and Kiyoshi Kuramoto , *Department of Cosmosciences, Hokkaido University, Sapporo 060-0810, Japan, (ftakashi@ep.sci.hokudai.ac.jp).*

Introduction: From the masses and orbital radii of the present solar planets, distribution of the surface density of solid component (dust) $\Sigma_{\rm d}$ in the solar nebula is expected to be $\Sigma_{\rm d} \propto r^{-3/2}$, ranging up to distance $r \simeq 40$ AU from the proto-sun [1]. The power law index is consistent with those of the "minimum mass extrasolar nebula" models reconstructed from extrasolar planetary systems, $p = 2.0 \pm 0.5$ (where $p \equiv -\partial \ln \Sigma_{\rm d}/\partial \ln r$) [2]. In contrast, observed T Tauri disks typically spread several hundred AU and have $0 < p < 1$, respectively [3].

Dust particles experience the global radial redistribution associated with turbulent diffusion and inward drift induced by gas drag in an accreting protoplanetary disk. These processes are potentially responsible for the discrepancy in the surface density distribution. However, simulations on the redistribution processes for growing dust aggregates by collisional agglomeration up to planetesimal size ($\sim$ km) have not reproduced the $3/2$ power law distribution [4,5].

Collision experiments of dust aggregates show that sticking hardly occurs above a threshold collision velocity [6]. This suggests that the collisional agglomeration is possibly suspended at the aggregate size much smaller than m-size. A simulation on the redistribution for dust aggregates with a uniform size (taken the typical size of chondrules) has found that the dust aggregates generate local enhancement in the dust surface density as they drift inward [7]. Although this effect might be capable of reproducing the dust surface density distribution with $p \simeq 3/2$, whether the distribution evolves flatter or steeper depends on the surface density and temperature profile of the disk gas.

In this study, we assume a uniform threshold velocity for sticking, instead of a uniform aggregate size, throughout the disk and examine the redistribution process of icy dust aggregates in the present outer planet region. As a result, we find that dust aggregates are redistributed maintaining $p \simeq 3/2$, almost independently of the initial distribution of dust and even the structure of the disk gas.

Analytical Study: In the turbulent disk gas, the collision velocity between the dust aggregates with radius d_1 and d_2 (1 m $\gg d_1 \geq d_2$) is given by [8];

$$v_{\rm coll} \simeq \sqrt{\frac{t_{\rm stop}(d_1)}{t_{\rm K}}} v_{\rm turb} \ , \qquad (1)$$

where $t_{\rm stop}(d_1)$ is the stopping time of the aggregate with d_1, $t_{\rm K}$ is the local Kepler time, and $v_{\rm turb}$ is the velocity of the largest turbulent eddy, respectively. According to the α viscosity model [9], $v_{\rm turb} = \sqrt{\alpha} c_{\rm s}$ where α is the non-dimensional parameter and $c_{\rm s}$ is the sound speed of the disk gas. $t_{\rm stop}$ and $v_{\rm coll}$ increase with the size of aggregates. As the collision velocity of an aggregate finally reaches the critical velocity for sticking $v_{\rm crit}$, further growth is suspended. The largest aggregate size $d_{\rm max}$ achieved by the collisional agglomeration is therefore described by;

$$\sqrt{\frac{t_{\rm stop}(d_{\rm max})}{t_{\rm K}}} v_{\rm turb} = v_{\rm crit} \ . \qquad (2)$$

Here we assume that the largest aggregate size is determined locally. This assumption may be reasonable if the growth time $t_{\rm grow}$ is shorter than the drift time $t_{\rm drift}$ for the largest aggregates;

$$\frac{t_{\rm grow}}{t_{\rm drift}} = \frac{4\rho_{\rm bulk}|v_r|d_{\rm max}}{\rho_{\rm d} v_{\rm coll} r} < 1 \ , \qquad (3)$$

where $\rho_{\rm bulk}$ and $\rho_{\rm d}$ are the bulk and the spatial densities of the aggregates and v_r is the radial drift velocity. Actually, the above condition is well satisfied in usual disk environments. A numerical simulation on the time-dependent collisional growth including the effect of fragmentation shows the tendency that the larger aggregates occupy the larger part of the total dust mass [10]. Thus, we take $d_{\rm max}$ as the typical aggregate size at each location in the disk.

Time variation of the surface density of icy dust aggregates is governed by;

$$\frac{\partial \Sigma_{\rm d}}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(\Sigma_{\rm d} v_r r\right) - \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\Sigma_{\rm d} D r^{1/2}\right)\right] = -S\delta(r - r_{\rm ice}) \ , \qquad (4)$$

where D is the turbulent diffusivity. The RHS represents the net sink associated with sublimation and condensation of icy component at the ice line with $r = r_{\rm ice}$, where S is the net sublimation rate and $\delta(x)$ is the Dirac delta function, respectively. Using Eq. (2), the radial drift velocity is given by;

$$v_r \simeq -\frac{t_{\rm stop}(D)}{t_{\rm K}}\frac{c_{\rm s}^2}{v_{\rm K}} = -\frac{v_{\rm crit}^2}{\alpha v_{\rm K}} \ , \qquad (5)$$

where $v_{\rm K}$ is the local Kepler velocity. Note that v_r is no longer dependent on the structure of the disk gas (cf., [7]). The diffusion term in Eq. (4) is negligible when $|v_r| \gg |v_{\rm diff}|$, where $v_{\rm diff} \simeq -D/r = -\alpha c_{\rm s}^2/v_{\rm K}$ assuming D equals the turbulent viscosity ν. If this is the case, in the outer part of the disk, Eq. (4) is rewritten as;

$$\frac{\partial \Sigma_{\rm d}}{\partial t} \simeq -\frac{1}{r}\frac{\partial}{\partial r}\left(\Sigma_{\rm d} v_r r\right) \ . \qquad (6)$$

v_r is proportional to $r^{1/2}$ if α and $v_{\rm crit}$ are constant throughout the disk. Thus, for $p < 3/2$ the RHS of (6) is positive and the surface density distribution becomes steeper. In the same manner, for $p > 3/2$ the RHS is negative and the distribution becomes flatter. Therefore, the dust surface density is expected to evolve with time maintaining $p \simeq 3/2$.

Numerical Simulation: Next, we perform numerical simulation on our redistribution model to clarify the time scale of the evolution, parameter dependency, etc. In addition to Eq. (4), we solve the equations for time variation of the surface density of the disk gas and water vapor which sublimates from icy dust at the ice line;

$$\frac{\partial \Sigma_{\rm g}}{\partial t} - \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\Sigma_{\rm g} D r^{1/2}\right)\right] = 0 \quad (7)$$

$$\frac{\partial \Sigma_{\rm v}}{\partial t} - \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\Sigma_{\rm v} D r^{1/2}\right)\right] = S\delta(r - r_{\rm ice}) \quad (8)$$

The temperature of the disk midplane, which affects the intensity of turbulence and location of the ice line, is calculated with a simple radiative transfer model including irradiation form the central star and viscous dissipation as heat source [5].

We consider a disk with the initial mass and dust-gas ratio of $M_{\rm disk} = 0.1 M_\odot$ and 0.01 which surrounds the central star with mass, radius and surface temperature of 1 $M_\odot$, $2R_\odot$ and 4000 K, respectively. The bulk density of icy aggregates is 2 g cm^{-3}. The initial radius $r_{\rm init}$ and power law index $p_{\rm init}$, α and $v_{\rm crit}$ are parameters ranging 100–1000 AU, 0.5–1, 10^{-2}–10^{-3} and 0.3–3, respectively.

Results & Discussion: Figure 1 shows the result for $r_{\rm init} = 300$ AU, $p_{\rm init} = 1.0$, $\alpha = 3 \times 10^{-3}$ and $v_{\rm crit} = 1$ m s^{-1}. In each panel, the discontinuity around 3 AU is the location of the ice line. After the onset of disk accretion, the dust surface density begins to decrease from the outermost region of the disk, associated with inward drift of dust. At the same time, the inward drift enhances the solid surface density at the inner region of the disk, in a similar manner as [7]. The power law index increases with time and $p \simeq 3/2$ is achieved within 10^6 yr at the region ranging from several to several tens AU. p is almost kept constant until $\sim 2 \times 10^6$ yr and then becomes steeper because of the arrival of the aggregates which initially existed at the outer edge of the disk, where the surface density distribution was cut off.

Difference in $p_{\rm init}$ affects the surface density profile during only the first $< 10^6$ yr. Thus, the $-3/2$ power law distribution is robustly reproduced after the initial relaxation period. The time scale of the redistribution depends on $r_{\rm init}$, α and $v_{\rm crit}$. The duration during which $p \simeq 3/2$, τ, is simply estimated as the drift time at the initial outer edge of the disk, $r_{\rm init}/v_r(r_{\rm init})$. Using Eq. (5), its dependency on the three parameters is;

$$\tau \simeq 10^6 \left(\frac{r_{\rm init}}{100\ {\rm AU}}\right)^{1/2} \left(\frac{\alpha}{3\times10^{-3}}\right) \left(\frac{v_{\rm crit}}{1\ {\rm m\ s^{-1}}}\right)^{-2} {\rm yr}, \quad (9)$$

which is comparable of the lifetime of T Tauri disks [11].

We conclude that the $-3/2$ power law distribution of solid component in the solar system would be simply and plausibly reproduced as a consequence of the radial drift of small dust aggregates, which have a uniform threshold velocity for sticking ~ 1 m s^{-1}, in an accreting protoplanetary disk.

References: [1] Hayashi, C. (1981) *PThPS* 70, 35–53. [2] Kuchner, M. J. (2004) *ApJ* 612, 1147–1151. [3] Kitamura, Y. et al. (2002) *ApJ* 581, 357–380. [4] Stepinski, T. F. & Valageas, P. (1997) *A&A* 319, 1007–1019. [5] Ciesla, F. & Cuzzi, J. N. (2006) *Icarus* 181, 178–204. [6] Blum, J. (2004) in *Astrophysics of Dust*, 369–391. [7] Youdin, A. N. & Shu, F. H. (2002) *ApJ* 580, 494–505. [8] Weidenschilling, S. J. (1984) *Icarus* 60, 553–567. [9] Shakura, N. I. and Sunyaev, R. A. (1973) *A&A* 24, 337–355. [10] Dullmond, C. P. & Dominik, C. (2005) *A&A* 434, 971–986. [11] Haicsh, K. E. et al. (2001) *ApJL* 553, 153–156.

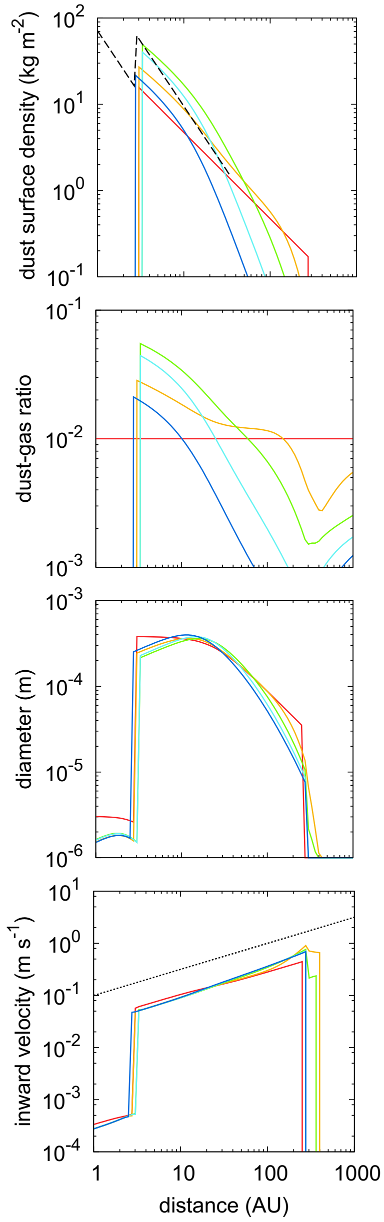

Figure 1: Results for $r_{\rm init} = 300$ AU, $p_{\rm init} = 1.0$, $\alpha = 3 \times 10^{-3}$ and $v_{\rm crit} = 1$ m s^{-1}. In each panel, the red, orange, green, light blue and blue curves represent $t = 0$, 3×10^5, 10^6, 2×10^6 and 3×10^6 yr, respectively. The dashed curve in the panel of dust surface density is that of the minimum mass solar nebula [1]. The dotted line in the panel of the inward drift velocity represents $v_r \propto r^{1/2}$.