ORBITAL CONFIGURATION SENSITIVITY OF AN ALTIMETRIC ORBITER FOR A EUROPA SOLID TIDE DETECTION MISSION. Stefano Casotto, Sebastiano Padovan and Massimo Bardella. Department of Astronomy, University of Padua, Padua, Italy (Stefano.Casotto@unipd.it), Center for Space Studies (CISAS), University of Padua, Padua, Italy.

Introduction. The aim of this study is a preliminary evaluation of the ability of a single Europa orbiting platform equipped with an altimeter to determine the Love parameters \( h_2 \) and \( k_2 \) of this Jovian satellite and to possibly discriminate between different models of Europa’s interior. This has been accomplished by simulating geocentric ranging to the spacecraft and altimeter measurements to the surface of Europa and by estimating relevant dynamical and geophysical parameters for several different inclinations of the orbit. The parametrization with inclination provides a preliminary answer to the design of an effective orbital configuration.

Dynamical Model. The approach followed in this study makes use of numerical integration to generate the orbit of an orbiter around Europa, modelled as an extended body with gravitational field \( U \) and surface gravity \( g \), perturbed by Jupiter, Io, Ganymede, Callisto, and the Sun treated as point masses. The equations of motion are therefore

\[
\ddot{\mathbf{r}} = \frac{\partial U}{\partial \mathbf{r}} + \sum_p GM_p \left( \frac{\Delta}{\Delta^3} \mathbf{r}_p \right),
\]

where the potential of Europa is modelled as

\[
U = \frac{GM_E}{r} \left[ 1 + \sum_{n=2}^{5} \left( \frac{R_E}{r} \right)^n \times \left( \begin{array}{c} C_{nm} \cos m\lambda + \Delta C_{nm} \\ S_{nm} \sin m\lambda + \Delta S_{nm} \end{array} \right) P_n^m \left( \sin \theta \right) \right].
\]

The Stokes coefficients \( C_{nm}, S_{nm} \) account both for the static field of Europa \( E \) and its time-variable part \( T \) due to the tidally redistributed mass:

\[
\overline{C}_{nm} = i\overline{S}_{nm} = \left( C_{nm} + \Delta C_{nm} \right) - i \left( S_{nm} + \Delta S_{nm} \right),
\]

where in terms of the distance \( r_p \), longitude \( \lambda_p \), latitude \( \phi_p \) of the perturbers

\[
\Delta C_{nm} - i\Delta S_{nm} = \frac{k_m}{2} \sum_n \left( \frac{R_E}{r_p} \right)^{n+1} \left( \frac{GM_E}{r_p} \right)^n P_n^m \left( \sin \phi_p \right) e^{i n \phi_p}.
\]

Kinematical Model. The equations of motion of the Europa orbiter and its associated partial derivatives have been numerically integrated in the J2000, Europa-centered reference frame. The positions of the perturbing bodies and of the Earth’s center of mass are obtained from the JPL planetary and natural satellite ephemerides.

Tide Model. To model tides on Europa we used the classical Love formulation:

- Surface radial deformation: \( s_i = h_i \sum_p \left( V^P_2 / g \right). \)
- Tidal Potential: \( \Delta U = k_i \sum_p V^P_2. \)

\( V^P_2 \) is the second degree term of the tide-generating potential of perturber \( P \). Values for \( h_2 \) and \( k_2 \) are taken from \cite{2} and are listed here in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>( h_2 )</th>
<th>( k_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.26</td>
<td>0.261</td>
</tr>
<tr>
<td>B</td>
<td>1.25</td>
<td>0.259</td>
</tr>
<tr>
<td>C</td>
<td>1.16</td>
<td>0.241</td>
</tr>
</tbody>
</table>

Table 1 – Europa models, \cite{2}.

Observation Data Types.
- Geocentric range measurements with \( \sigma = 3 \)cm;
- Altimetry measurements with \( \sigma = 10 \)cm;

Detecting the Ocean on Europa. The spacecraft orbits in a highly perturbed environment caused by the nearby presence of the large mass of Jupiter. Small differences in initial conditions and/or dynamical model parameters—essentially \( k_2 \)—quickly lead to divergence between the true and the nominal orbits. Nevertheless, the ratio \( h_2/k_2 \) for models with and without ocean has a value of \( \sim 4.5 \) and \( \sim 2 \), respectively \cite{1} (see values in Table 1). Using this feature it is possible to discriminate among the two types of models, as reported in Table 2. In our simulations the true value of this ratio is recovered for all inclinations (barring divergence), but the higher the inclination, the stronger the indication of the presence of an ocean.

The large error on the low inclination orbits is driven by the uncertainty of \( k_2 \) (see next section).

Table 2 – Estimating the ratio \( h_2/k_2 \).

<table>
<thead>
<tr>
<th>2-day orbital arc</th>
<th>( i = 20^\circ )</th>
<th>( i = 70^\circ )</th>
<th>( i = 90^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_2/k_2 ) (true value)</td>
<td>4.83</td>
<td>4.83</td>
<td>4.83</td>
</tr>
<tr>
<td>( h_2/k_2 ) (nominal value)</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
</tr>
<tr>
<td>( h_2/k_2 ) (estimated)</td>
<td>Diverg.</td>
<td>4.83</td>
<td>4.83</td>
</tr>
<tr>
<td>( \sigma (h_2/k_2) )</td>
<td>Diverg.</td>
<td>0.99</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Errors on $k_2$ and $h_2$. Figures 1 and 2 show the formal estimation errors on $k_2$ and $h_2$ as a function of the length of the measurement arc. Each curve refers to a different orbital inclination. Both for $h_2$ and $k_2$ there is first a rapid decrease of the error, while extending the arc beyond 20 days leads to less marked improvement. In the estimation procedure the initial values of the Love numbers were changed by 20% to 30% with respect to the truth values. Low inclination orbits are more sensitive to $h_2$, as shown in Figure 2. High inclination orbits are more sensitive to $k_2$ than those at lower inclinations, as Figure 1 shows. The formal estimation error on $k_2$ is highly nonlinear as a function of inclination since a rapid decrease is followed by an essentially flat behaviour for inclinations higher than 70° (the curve—not shown—corresponding to a 45° inclination orbit would fall approximately halfway between the 20° and the 70° curves).

Discriminating Similar Model. Differences of an order of magnitude in the ice shell thickness in the ocean models for Europa shown in Table 1 do not appreciably affect Love numbers. This means that discriminating between models of different ice shell thickness may be unfeasible on the basis of the tools of space geodesy adopted here. Our simulations show that the estimation of the Love parameters from observations generated using the values of model A, while initializing their nominal values at either model B or C, is correctly achieved. However, the associated errors, while decreasing with the length of the observation arc, tend to level off at some finite value, as already noted in the previous section. This value effectively constitute a threshold beyond which different models cannot be discriminated.

Figure 3 refers to the case in which model A (1-km ice crust) is used for the truth values and model C (100-km ice crust) for the nominal values. The 70° inclination orbit used in this simulation immediately recovers the true value of $k_2$. Marginal discrimination is reached after 20 days also for $h_2$. This means that it could at least be possible to determine which component, ice or water, is dominant in the outer shell of Europa but finer discrimination is not possible.

Conclusions. Numerical simulations of the estimation of the Love parameters from altimeter and ground tracking measurements of a Europa orbiter have shown that it is possible to detect the presence of a subsurface ocean from the tidal deformations of the satellite surface and the orbital perturbations within a few days of observations. Some discrimination can be achieved as to the depth of the ice crust within several weeks of observation, the balance in favor of dominance by water or ice being generally more reliable. The orbital inclination of the Europa orbiter should be selected in the range of mid- to high-inclination, near polar orbits not been required.