YORP SENSITIVITY TO SHAPE AND SHADOWING. D.J. Scheeres, *U. Colorado, Boulder (scheeres@colorado.edu)*, S. Mirrahimi, *U. Michigan, Ann Arbor*, R.W. Gaskell, *PSI*.

We present a new secular theory of the YORP effect on the rotation state of an asteroid in terms of a set of geometry-dependant coefficients. The theory can be used to study the sensitivity of YORP to the fine-scale shape and global shape of an asteroid, to shadowing, and to thermal inertia. The secular effect of YORP on the rotation state can be captured through a series of polynomials that are a function of the orbit obliquity and are scaled by terms involving the asteroid heliocentric orbit, the thermal parameter, and a characteristic scale factor for the asteroid. The motivation for this research is to investigate possible explanations of the non-detection of YORP for Itokawa at its last apparition 1, and to develop a more complete understanding of the role of shape and mass distribution in YORP.

The secular effect of YORP   In 2 the derivation of the YORP effect in terms of Fourier coefficients given in 3 is generalized to spherical harmonics coefficients to find a generic form for the secular equations for an asteroids rotation state as a function of its current spin rate, current obliquity, heliocentric orbit semi-major axis and eccentricity, a characteristic YORP coefficient, the asteroid thermal inertia (assumed uniform over the body) and a series of spherical harmonic coefficients that are only a function of the asteroid's shape. The torque from incident solar radiation can be modeled using a spherical harmonics expansion:

$$\frac{\mathbf{M}(R, \delta_s, \lambda_s)}{P(R)} = \sum_{l=0}^{\infty} \sum_{m=0}^{l} P_l^m(\sin(\delta_s)) \{ \mathbf{C}_{l,m} \cos(m\lambda_s) + \mathbf{D}_{l,m} \sin(m\lambda_s) \}$$

where the sun's location in the asteroid-fixed frame is specified by the latitude δ_s and the longitude λ_s, $P(R)$ is the solar radiation pressure at a distance R from the sun, $P_l^m(x)$ are the Associated Legendre Functions and we note that $\mathbf{C}_{l,m}$ and $\mathbf{D}_{l,m}$ are vectors with component directions x, y, and z, corresponding to the minimum, intermediate and maximum moment of inertia axes. Computation of these coefficients for a given asteroid shape is discussed in 2. Thermal inertia induces a delayed emission, which can be modeled as a reduction in the strength of the torque and by a lag angle ϕ_{lag} in the longitude.

The equations of rotational motion for the asteroid are averaged over one asteroid rotation and one asteroid revolution about the sun, assuming that the body is close to principal axis rotation. If we assume that the re-radiation of absorbed sunlight dominates the YORP effect, the secular equations take the form:

$$\dot{\omega} = \frac{g}{\sqrt{1 + 2\mu\sqrt{\omega} + 2\mu^2\omega}} A(\beta)$$

$$\dot{\beta} = \frac{g(1 - \beta^2)\beta}{\omega \left(1 + 2\mu\sqrt{\omega} + 2\mu^2\omega \right)} \left[B(\beta) + \mu\sqrt{\omega}D(\beta) \right]$$

with the asteroid rotation state defined by ω, the rotation rate of the asteroid, and $\beta = \sin(i)$, where i is the obliquity of the asteroid. The secular evolution of the right ascension of the asteroid rotation pole can also be described, but is not given here. The parameters of the secular equations, g and μ, are combinations of physical parameters:

$$g = \frac{G_1 r}{M a^2 \sqrt{1 - e^2}}$$

$$\mu = \frac{\sqrt{\varrho c_p \kappa}}{\sqrt{32} \varepsilon \sigma T_{eq}^3}$$

For the g parameter, G_1 is the solar radiation constant at one astronomical unit, r is the mean radius of the asteroid, M is the total mass of the asteroid, a is the semi-major axis of the asteroid's heliocentric orbit, and e is the eccentricity of the asteroid's heliocentric orbit. For the μ parameter, ϱ is the density of the asteroid surface layer, c_p is the specific heat, κ is the thermal conductivity, ε is the emissivity, σ is the Stefan-Boltzmann constant, and T_{eq} is the equilibrium temperature. The quantity $\sqrt{\frac{1}{2}\varrho c_p \kappa \omega}$ is the thermal inertia and is proportional to μ. The detailed solution we use was taken from the linearized solution to the surface heat transfer equations given in 4. The thermal lag of the asteroid's re-radiation of the absorbed solar photons can be computed from these parameters as $\tan(\phi_{lag}) = \frac{\mu\sqrt{\omega}}{1 + \mu\sqrt{\omega}}$.

The terms $A(\beta)$, $B(\beta)$ and $D(\beta)$ are analytic functions of β and can be expressed as: $A(\beta) = \sum_{k=0}^{\infty} a_k \beta^{2k}$, $B(\beta) = \sum_{k=0}^{\infty} b_k \beta^{2k}$, $D(\beta) = \sum_{k=0}^{\infty} d_k \beta^{2k}$, where the coefficients a_k, b_k, and d_k can be computed analytically from the spherical harmonic coefficients. Specifically a_k is a function of $C_{2t,0,z}$, b_k is a function of $C_{2t,1,x}$ and $D_{2t,1,y}$, and c_k is a function of $-C_{2t,1,y}$ and $D_{2t,1,x}$. Note that if we consider rotation in the opposite sense the coefficients undergo the following symmetry transformations:

$$a_k' = -a_k$$
$$b_k' = -b_k$$
$$c_k' = c_k$$
$$d_k' = -b_k + c_k$$

Thus we note that there is a non-symmetric change in the obliquity evolution if the asteroid spins in the opposite direction.

Shape effects on secular evolution   If the asteroid has a non-uniform mass distribution its center of mass may be displaced from its center of figure. Such a displacement along the z-axis will create no change in the spin rate evolution, while displacements along the x or y axes will impact the rate evolution. For the obliquity, displacements in a given direction have no impact on coefficients from that direction, but can have a large impact on coefficients from orthogonal directions.

For the rotation rate evolution, we see that only the constant torques about the body's maximum moment of inertia influence the change in rotation rate. The effect of thermal inertia is only on the overall magnitude of the re-radiated photons.

For the obliquity evolution we see more complex behavior. Over one rotation and revolution all of the torques average to zero except those of order one parallel to the sun's instantaneous location. In other words, as the sun moves in the body fixed frame, the torques which contribute to the secular evolution of the body's obliquity are those which cause the body's angular momentum to rotate about the sun-line. The function $B(\beta)$ ideally represents the instantaneous reemission of solar photons, while the function $D(\beta)$ ideally represents the reemission of solar photons with a lag angle of $90°$. For the true system reemission is distributed between these, with the B function multiplied by $\cos \phi_{lag}$ and the D function multiplied by $\sin \phi_{lag}$. From geometric considerations alone, we expect the c_k, and hence d_k, terms to be larger than the b_k obliquity terms, often by orders of magnitude or more. This is due to the classical shape of an asteroid, with longest axis about the x-axis and shortest about the z-axis. The coefficients that contribute to the D function generally have a much larger moment arm for the incident radiation pressure to act on. This is especially true for a strongly oblate asteroid such as 1998 KW4 alpha, shown in Fig. 1. Thus, although a non-zero thermal inertia will decrease the magnitude of the YORP effect, it's effect can still dominate the obliquity evolution through the $D(\beta)$ function.

When considering precision computations of the YORP coefficients, the resolution of the asteroid's shape model becomes very important, as the physical size scale at which radiometric effects are operative range down to the sub-micron level. Shadowing effects can also become very important and yield significant changes in the YORP coefficients. In Fig. 2 the $A(\beta)$ function for the asteroid Itokawa is shown computed at different shape model resolutions, with and without shadowing. The quoted resolutions are just approximate. Due to this sensitivity, future research should model the interaction of photons with the fine-scale structure of the surface using more sophisticated models than have been used in the YORP literature.

[1] Kitazato, K., et al. (2007) "25143 Itokawa: Direct Detection of the Current Decelerating Spin State due to YORP Effect," DPS Meeting, Orlando, FL Abstract 05.05.
[2] D.J. Scheeres and S. Mirrahimi (2008) "Rotational Dynamics of a Solar System Body Under Solar Radiation Torques," *Celestial Mechanics and Dynamical Astronomy*, "," in press.
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[4] D.P. Rubincam (1995) "Asteroid orbit evolution due to thermal lag," *Journal of Geophsical Research* 100(E1), 1585-1594.
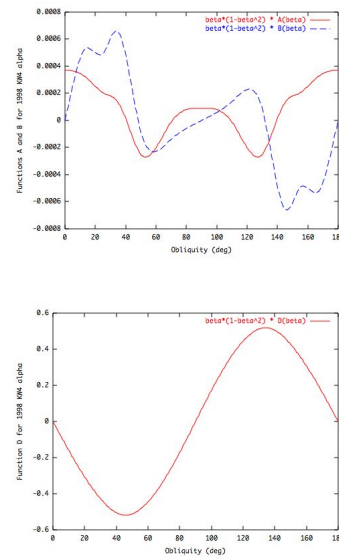
Figure 1: Functions $A(\beta)$, $\beta(1 - \beta^2)B(\beta)$ and $\beta(1 - \beta^2)D(\beta)$ as a function of β for asteroid 1998 KW4 alpha. Note that $D(\beta)$ is 4 orders of magnitude larger than $B(\beta)$ due to the "flying saucer" shape of the asteroid.
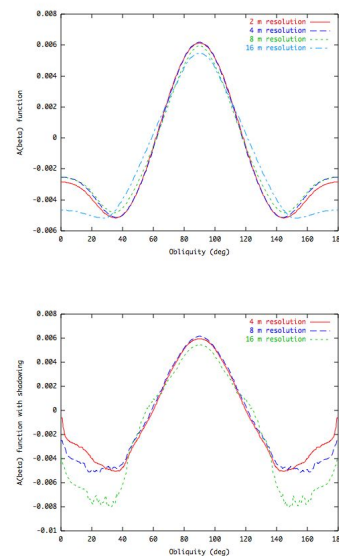

Figure 2: Function $A(\beta)$ as a function of shape model resolution, with and without shadowing.