**HEAT AND DRAG COEFFICIENTS FOR REENTRY OF IMPACT EJECTA.** H. J. Melosh and T. J. Goldin, Lunar and Planetary Lab and Geosciences Department, Universiy of Arizona, Tucson AZ 85721 (jmelosh@lpl.arizona.edu).

**Atmospheric Entry of Impact Ejecta:** Distal ejecta particles from large impacts on the Earth or other planets possessing atmospheres lose most of their kinetic energy in the atmosphere before deposition on the surface. Such particles are generally small: Most of the distal K/Pg ejecta was in the form of glassy spherules about 300 microns in diameter [1], similar in size to impact spherules deposited by distant impacts in the Archean [2]. The dark parabolae surrounding fresh Venusian craters imply particle sizes ranging from about 1 micron to 1 cm in diameter [3] ejected to ranges of several thousand km from the impact site. Entry velocities of such ejecta range from planetary escape velocity (11.2 km/sec for the Earth) down to just enough speed to heave ejected particles out of the atmosphere (about 1.4 km/sec on the Earth).

Once fast ejecta partiles enter the atmosphre, they decelerate to terminal velocity and then drift downward at low speed, perhaps being entrained in density currents, before deposition on the surface. Deceleration in the upper atmosphere initially occurs at very high speed, during which aerodynamic friction strongly heats the particles, a process first analyzed by Whipple [4]. Intense thermal radiation from the hot particles, however, keeps them relatively cool: This cooling permits the collection of relatively unscathed interplanetary dust particles in the Earth's atmosphere [5, 6].

**The Aerodynamic Problem:** Quantitative modeling of the entry and deposition of impact ejecta requires adequate mathematical expressions for the drag and frictional heating of small particles over a wide range of both velocity and atmospheric density. Conventional equations for these quantities assume that atmospheric gases can be treated as a continuum. However, small particles in the upper atmosphere, where most of the high speed deceleration occurs, are typically much smaller than the mean free path of gas molecules. As the particles settle through the atmosphere the density increases and it becomes necessary to treat the particles' motion through the intermediate regime of semi-continous gas down to the continuum limit of low speed Stokes flow.

Over the past year we have been engaged in modeling the entry of distal K/Pg ejecta into the Earth's atmosphere. During this investigation we found that there are no generally accepted expressions for drag and heat transfer coefficients over the range of conditions we are encountering in our models. Atmospheric entry models by Whipple [4] and Opik [7], however revolutionary they were at the time, have long been superceded by directly-tested models in the aerospace engineering literature [8].

We searched the aerospace literature for analytic expressions of drag coefficients for spheres over the range of conditions relevant for the entry and deposition of impact ejecta. Many formulae exist that cover only part of the total range, but for modeling purposes we needed only one, or a few, formulae that smoothly transition into one another. As is conventional in fluid mechanics, these expressions are formulated in terms of dimensionless numbers characterizing the flow. At low speeds the Reynolds number $Re$ alone suffices, $Re = Lv/\nu$, where $L$ is a relevant dimension (the particle diameter in our case), $v$ the relative velocity of the particle and the atmospheric gases and $\nu$ is the kinematic viscosity of the gas. However, at high speeds, the Mach number $M = v/c$, where $c$ is the speed of sound in the gas, is also important. A related factor is the Knudsen number, $Kn = M/Re$, equal to the ratio between the particle diameter and the mean free path of molecules in the gas.

**Drag Coefficient:** The most comprehensive drag equation that we found was formulated to express the motion of small solid particles in the hot gases streaming out of the nozzle of a solid fuel rocket [9]. However, this equation for the drag coefficient incorporates transcendental functions that become singular in limiting cases and in any case are very slow to evaluate numerically. We thus substituted simpler functions that behave well numerically and nevertheless agree with Crowe's equations (and with the extensive data sets he cites!) to a precision of better than 0.1%. Another problem that arises is that Crowe expressed his drag coefficient in terms of a continuum limit expression that he does not define explicitly. In other papers in the aerospace literature [10], this limiting expression is taken to be the Stokes flow limit at vanishing Reynolds number. However, using this identification we were unable to reproduce Crowe's low Reynolds number fit in his Fig 1. Instead, we found that using a continuum limit that incorporates low Reynolds number deviations from Stokes flow [11] will, in fact, yield the correct fit.

Summarizing our expression for an analytical drag coefficient valid over the full range of conditions describing the entry of small ejecta particles, we have:

$$C_D = 2 + [C_{inc} - 2]\, e^{-\left[\frac{3.07\sqrt{\gamma}\, M\, G(Re)}{Re}\right]} + \frac{H(M)\, e^{\frac{-Re}{2M}}}{\sqrt{\gamma} M}$$

where γ is the gas specific heat ratio and the incompressible limit drag coefficient is:

$$C_{inc} = \frac{24}{Re}(1 + 0.15 Re^{0.687})$$

and the two auxiliary functions *G(Re)* and *H(M)* are given by:

$$\mathrm{Log}_{10}[G(Re)] = \frac{2.5\left(\frac{Re}{312}\right)^{0.6688}}{1 + \left(\frac{Re}{312}\right)^{0.6688}}$$

$$H(M) = \frac{4.6}{1+M} + 1.7\sqrt{\frac{T_p}{T_g}}$$

where $T_p$ is the temperature of the particle and $T_g$ is the temperature of the gas in the free field.

**Heat Transfer from Hypersonic to Stokes Regime:** In a similar manner we sought improved expressions for the rate at which heat is transferred both to the entering particle and the gas. Because we are using a numerical hydrodynamic simulation of particle flow, we require formulae that express both the heat transferred to the entering particle, and to the gas itself. In the aerospace literature, however, the focus is on heat transfer to the particle and the heat transfer coefficient, the Nusselt number *Nu*, is given a form that is awkward for inclusion in the full energy conservation equations in a hydrodynamic computer code. We thus separated the standard equations into two more convenient terms.

The total kinetic energy lost by an entering ejecta particle is given by $vF_D$, where $F_D = 0.5\rho_g A C_D v^2$ is the drag force. The rate of heat transfer $Q$ to a sphere, which concatenates both friction and heat conduction, is defined in terms of the Nusselt number by $Q = \pi L k_g Nu(T_r - T_p)$, where $k_g$ is the thermal conductivity of the gas and $T_r$ is the "recovery temperature" of the particle [12]. This is the temperature that, if attained by the particle, is such that no heat is exchanged with the gas (it is sometimes called the "adiabatic wall temperature"). It is computed from the "recovery factor" $r$, defined in

$$(T_r - T_g) = r(T_s - T_g) = \frac{\gamma - 1}{\gamma} M^2 T_g r$$

where $T_s$ is the stagnation temperature of the gas. The recovery factor $r$ depends on the Prandtl number, but its value is within 10% of 1 for air [12], even in free molecular flow [13].

We decomposed the heat transfer equation, proportional to $(T_r - T_p)$, into a factor proportional to $(T_r - T_g)$ and another proportional to $(T_p - T_g)$. The first factor represents heat from compression and friction with the gas, while the second is the conventional heat exchange between two bodies of differing temperature. This allows us to define a friction factor $\alpha$, the fraction of the total energy loss that is transferred into the particle, given by

$$\alpha = \frac{8}{\gamma}\left(\frac{Nu}{Re\ Pr}\right)\frac{r}{C_D}$$

Of course, the fraction $(1-\alpha)$ of the kinetic energy loss is transferred to the gas.

The Nusselt number itself is given by [14, 15]

$$Nu = \frac{Nu_C}{1 + 3.42 \frac{M'}{RePr} Nu_C}$$

where the continuum limit of the Nusselt number is

$$Nu_C = 2 + 0.459 Re^{0.55} Pr^{0.33}$$

Note that the equation for Nu contains a factor *M'*, which is not the Mach number. We found that this equation, due to Kavanau [15], does not extrapolate correctly to the rarefied gas limit, nor is it correct for very high Mach number. However, we found a correction for this formula by using

$$M' = \frac{M}{1 + 0.428\left(\frac{\gamma+1}{\gamma}\right)M}$$

Using this replacement, the equation behaves correctly in both low and high density limits, and over the entire range of velocities, from Stokes regime to hypersonic.

These equations were compared with the results from the Whipple model, showing that that Whipple's estimated temperatures are typically a few hundred K too high for entry of a 0.5 mm particle at 8 km/sec.

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