A FAST*ER* MODEL FOR COAGULATIVE GROWTH IN NEBULAR AND SUBNEBULAR ENVIRONMENTS. P. R. Estrada, *Carl Sagan Center, SETI Institute, Mountain View CA 94043, USA, (Paul.R.Estrada@nasa.gov)*, J. N. Cuzzi, *NASA Ames Research Center, Moffett Field CA 94035, USA (Jeffrey.Cuzzi@nasa.gov)*.

Introduction: The conditions under which solid body formation is initiated in protoplanetary and protosatellite disks remain poorly understood. With the continued discovery of extrasolar planets, an emphasis is placed on the need to understand the formation of not only our own solar system, but of these numerous strange and diverse systems as well. In particular, many of these giant extra solar planets almost certainly have the potential for harboring regular satellite systems.

At the forefront still remains the basic question of how growth occurs from dust which is entrained in the gas, to larger, relatively immobile planets and satellites. The key properties of protoplanetary nebulae and subnebulae remain controversial, yet grain growth from dust to larger agglomerates, and from planetesimals (satellitesimals) to planets (satellites) must have occurred in some manner. While numerous studies of N-body dynamics have addressed the growth of terrestrial planets from asteroid-size planetesimals (e.g., [1,2]), it is the transition from agglomerates to planetesimals which continues to provide the major stumbling block in planetary origins [3,4,5]. Furthermore, no such efforts have thus far been attempted for protosatellite disks [6].

Theoretical approaches to the study of dust coagulation normally involve solving the cumbersome collisional coagulation equation given here in its integro-differential form [7]

$$\frac{df}{dt} = -\int_0^\infty K(m,m')f(m,t)f(m',t)\,dm' + \frac{1}{2}\int_0^m K(m-m',m')f(m-m',t)f(m',t)\,dm', \quad (1)$$

where $f(m,t)$ represents the particle distribution (here we take f to be the particle number density per unit mass), and the collisional kernel $K(m,m')$ for particles of mass m and m' typically includes particle cross-sectional area, particle-to-particle relative velocities, and some sort of sticking efficiency. However, the numerical solution to Eq. (1) at every time step, for every particle size, and at every spatial location is computationally expensive, and serves as the primary bottleneck in running evolutionary models over long periods of time. Therefore, seeking means by which we may overcome this computational burden is desirable. To this end, we report herein on the development of a nebula evolution model which employs a mathematical scheme that makes the full-scale solution to the problem of dust coagulation less prohibitive.

Methodology: We have recently developed a much faster way to calculate particle growth in turbulent and non-turbulent nebulae than current numerical models of coagulation. The new model uses a finite number of moments M_p (integral or fractional) of the particle size distribution [8]. This approach is particularly useful when only general properties of the distribution (e.g., opacity, largest particle size), and their time evolution, are needed. This "moments method" approach is (in principle) entirely analytical, and has the advantage over Eq. (1) that one need only calculate the time rate of change of the moments as opposed to tracking the entire mass histogram.

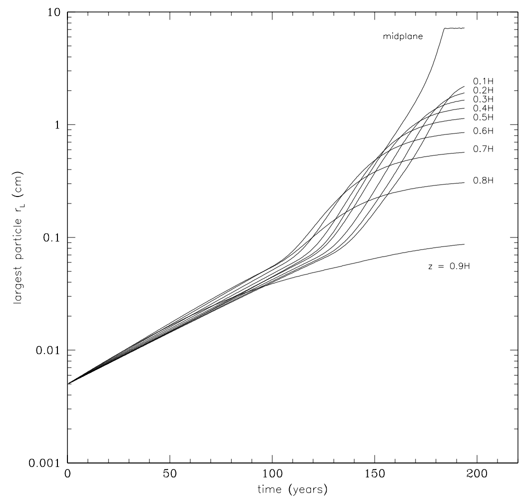


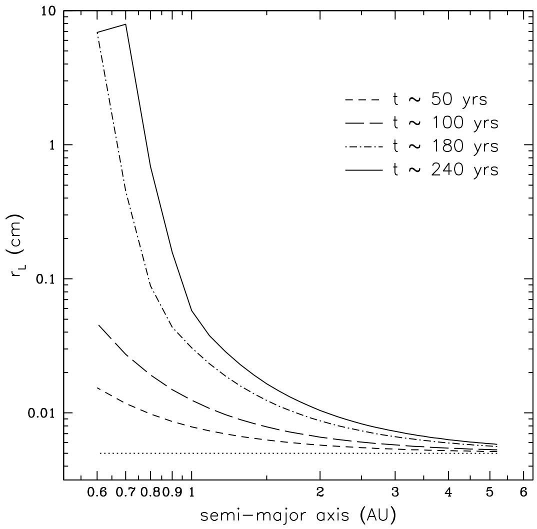
Figure 1: 2D simulation of particle growth at $R = 0.6$ AU. In the 2D model coagulative growth is followed in both vertical and radial cells. In the curve marked "midplane", the fragmentation barrier has been reached, and growth beyond this point is handled differently (Fig. 3), with the coagulation calculation providing a "creation rate" of sub-migrator material.

It is shown in [8] that Eq. (1) can be integrated to give the ODEs for the integral moments M_k ($k = 0, 1, 2, ..$)

$$\frac{dM_k}{dt} = \int_0^\infty\int_0^\infty \left[\frac{1}{2}(m+m')^k - m^k\right] \times K(m,m')f(m,t)f(m',t)\,dm\,dm', \quad (2)$$

which are valid for arbitrary choices of the collisional kernel. Ideally, one would like to be able to express the kernel K explicitly in terms of fractional/integral powers of m and m' so that the RHS of Eq. (2) can be expressed in terms of these; however, this is not easily done for the realistic kernels which include systematic (non-turbulent, that arise from gas pressure gradients [9]), and turbulence-induced relative velocities [10,11], as well as possibly a velocity dependent sticking coefficient [8].

Two different ways of utilizing the moments were devised to address this difficulty. Both approaches rely to different degrees on a simplifying assumption that the form of the particle size distribution is a powerlaw [8]. Such an assumption is not entirely unfounded. A number of detailed models [e.g., 3,12] have shown that powerlaw size distributions result, which have nearly constant mass per decade radius to an upper limit $m_L(t)$ which grows with time until a frustration limit ($m_L = m_*$) or "fragmentation barrier" is reached, and (under turbulent conditions, at least) growth stalls [5]. Similar trends are found by [4] in which different assumptions about collisional ejecta are



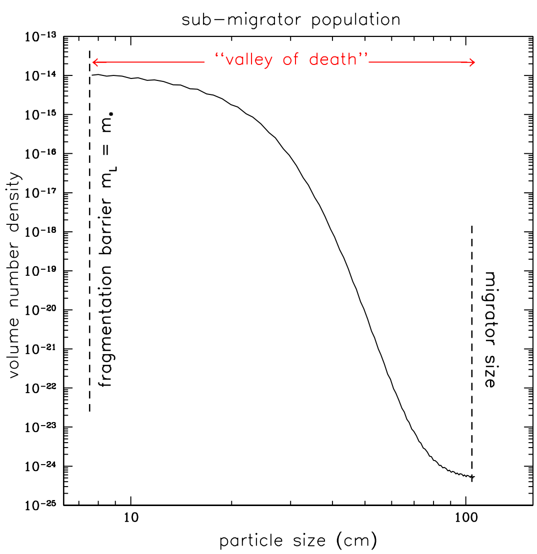


Figure 2: 1D simulated growth of the largest size particle as a function of semi-major axis for different times. For such a short run, most of the growth has occured close in while little has happened at large distances. The fragmentation size has been reached at 0.6 and 0.7 AU in the solid curve. $\Delta R = 0.1$ AU.

Figure 3: Simulation of growth beyond the fragmentation barrier (dashed line) where $m_L = m_*$ at $R = 0.6$ AU. The fragmentation size m_* is determined by the condition that the energy from a collision between a particle of size m_L and some $m \leq m_L$ exceeds the fragmentation energy Q_*. To the left of the dashed line, growth is by coagulation, whereas to the right growth proceeds by sweepup of dust and debris. The amount of material at larger sizes decreases steeply in this case because most of the material is destroyed before it can ever reach "migrator" size.

made. Our initial testing has found this treatment of growth up to the fragmentation barrier to be consistent with recent work by Brauer et al. [13] for the case in which additional effects such as radial drift of material are not included.

Global Model: Our intention here is to employ the moments method to obtain robust, quantitative results for space-and-time-dependent properties in global disk models, such as particle growth timescales and "typical" particle sizes that may be used in modeling efforts that are focused on the larger problem of planetesimal formation.

The 1D (radial) and 2D (radial and vertical) nebula evolution codes that we are currently developing use the methodology of [8] to treat the simultaneous growth and migration of particles up to the fragmentation barrier (see, Figures 1 and 2), where the fragmentation size depends on what one assumes for particle strength and choice of nebula parameters. We have included a variable sticking coefficient, allow for fragmentation, and take into account radial drift, vertical settling, and diffusion of particles. We have also incorporated a scheme that allows us to model the growth from beyond the fragmentation barrier, through a transition regime of "sub-migrators", to "migrators", which are defined as objects that have a low probability of disruption in the time it takes to drift across a radial bin (see, e.g., Fig. 3). Thus, a result of this model is that we can calculate the growth and disruption timescales for migrators at all locations in the nebula, rather than assuming growth times as has been done in other models [e.g, 14]. Moreover, we can keep track of the mass density of "migrators" that serves as feedstock for planetesimal growth. Models of the growth from migrators into relatively immobile planetesimals, which provide a sink for drifting and diffusing material, will also be described.

We will discuss the most recent results regarding formation timescales for migrators and planetesimals, differences between 1D and 2D treatments, and innovations designed to further speed up simulations, such as a timestep that is a function of the local orbital period. Interpolation and extrapolation schemes are used to keep track of material drifting between radial bins. Finally, we will discuss potential applications of the model in addition to modeling of the protoplanetary and protosatellite disks.

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