Titan's spin, gravity, and moments of inertia.

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Introduction Analysis of Doppler tracking data and radar images from the Cassini spacecraft have recently provided estimates of the low degree gravity field [1], and spin pole direction [2] of Titan. We examine implications of these measurements for the internal structure and rotational dynamics of that body. We derive separate estimates of the polar moment of inertia of Titan from the degree two gravity field, under the assumption of hydrostatic equilibrium, and from the spin pole direction, under the assumption of a fully damped spin-orbit configuration, or multi-frequency Cassini state. These estimates are quite different. We interpret the gravity-derived value as the actual moment of inertia of Titan, and the larger spin-derived value as an effective moment of inertia of a mechanically decoupled ice shell. This implies a sub-surface ocean, as the decoupling agent.

Gravity constraints: For a body in hydrostatic equilibrium and synchronous rotation, the imposed tidal and rotational potentials together induce changes in the mass distribution which are mainly manifest as degree two spherical harmonic coefficients in the gravitational potential [3]:

$$\begin{bmatrix} J_2 \\ C_{2,2} \end{bmatrix} = \frac{k_f\, q}{4}\begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

where the ratio of centrifugal and gravitational accelerations on the equator is

$$q = \frac{\omega^2 R^3}{3GM} = 1.315\times 10^{-5}$$

and k_f is a fluid Love number [4] or scale factor which relates the imposed and induced potentials. Observed values of the gravitational coefficients, from the Cassini tracking data [1], are consistent with this pattern, and have fluid Love numbers very close to 1. In contrast to the situation for the Galilean satellites [6,7,8,9] no *a priori* constraints were applied in deriving the coefficient estimates. Despite that, the inferred ratio of $J_2/C_{2,2}$ is very close to the hydrostatic value of 10/3.

If fluid Love numbers in the range (0.9-1.1) are used in the Darwin-Radau relation [5], we obtain an estimate of the polar moment of inertia

$$\frac{C}{MR^2} = \frac{2}{3}\left(1 - \frac{2}{5}\sqrt{\frac{4-k_f}{1+k_f}}\right) \approx 0.340 \pm 0.014$$

This value thus likely reflects the actual moment of inertia of Titan and suggests a reasonable degree of central condensation, though less than has been assumed in many theoretical models [10,11,12].

Spin pole constraints: The classical means of determining the moment of inertia of a planet, without hydrostatic assumptions, is via observation of the rate of spin pole precession. For a rapidly rotating body, this observation constrains the moment difference ratio H, where C H = C - (A+B)/2.

If the two gravitational potential coefficients

$$\begin{bmatrix} J_2 \\ C_{2,2} \end{bmatrix} MR^2 = \begin{bmatrix} C-(A+B)/2 \\ (B-A)/4 \end{bmatrix}$$

are also known, this provides 3 constraints on the 3 principal moments A < B < C, and they can all be determined. This is the means by which the moments of inertia of Earth [13] and Mars [14] are known.

A disadvantage for application of this strategy to a body like Titan is that the expected spin pole precession rate is very slow. A better approach, in such cases, is available if the spin pole is fully damped, since then the angular separation between spin and orbit poles is itself diagnostic of the moments of inertia. All that is required then is an accurate determination of the spin pole direction, rather than a determination of its rate of change. If the orbit pole precession rate is uniform, the damped spin pole will maintain a constant obliquity, or angular separation from the orbit pole, and will remain coplanar with the orbit pole and the invariable pole, about which the orbit pole is precessing. Such a configuration is known as a Cassini state [15,16], in honor of G.D. Cassini who realized in 1693 that the Moon behaves that way.

Titan does not quite satisfy the steady orbit precession criterion. The orbit precesses, with a period of 700 years and inclination of 0.28 degree, about Saturn's spin pole [17], but Saturn's spin pole also precesses, with a period of 1.87 million years and inclination of 26.7 degrees, about its own orbit pole [18]. However, the dynamical equivalent of a Cassini state configuration is easily extended to this multi-frequency situation.

The projection of the orbit pole unit vector onto the invariable plane can be represented as a complex scalar whose time evolution is given by a Poisson series

$$N[t] = \sum_j n_j\, Exp[I\,(f_j\, t + \gamma_j)$$

where n_j, f_j, γ_j are amplitude, rate, and phase for the j-th term. The linearized equation of motion for the complexified spin pole S is

$$\frac{dS}{dt} = -I\alpha(N - S)$$

where the rate parameter is

$$\alpha = \frac{3n}{2}\left(\frac{C - A}{C}\right) = \frac{3n}{2}\left(\frac{J_2 - 2C_{2,2}}{C/MR^2}\right)$$

The corresponding fully damped spin pole has the form [19, 20]

$$S[t] = \sum_j s_j Exp[I(f_j t + \gamma_j)]$$

with coefficients obtained from the orbit pole via

$$s_j = \left(\frac{\alpha}{\alpha + f_j}\right) n_j$$

In a multi-frequency version of the Cassini state, the spin and orbit poles are no longer coplanar with the invariable pole, as has been observed for Titan [2]. This is not necessarily evidence of failure to be in a fully damped state, but may simply reflect the more complex orbit pole dynamics.

The polar moment value required to match the observed spin pole orientation is

$$c \equiv C/MR^2 = 0.66$$

This is clearly in excess of the homogeneous spherical value of c = 2/5, but agrees well with the thin shell value of c = 2/3. It is thus plausibly interpreted as an effective moment of inertia of an outer ice layer which is mechanically decoupled from the deeper interior.

The problem of precessional coupling between Earth's fluid core and solid mantle has long been studied experimentally [21, 22], analytically [23, 24] and observationally [25, 26]. The situation at Titan is still in its infancy, but the system parameters are different enough from Earth that it is already informative.

Synchronous spin: The radar-derived spin rate has been reported to be faster than that required for a synchronous state [1] and the purported excess rotation rate claimed as evidence for angular momentum exchange between the atmosphere and solid body, with implication of a decoupled ice shell [27]. Though this latter point is in basic agreement with our interpretation of the gravity and spin pole results, as outlined above, we take issue both with the proposed mechanism and with the premise of a super-synchronous spin rate.

Much of Titan's atmosphere is observed to be super-rotating relative to the solid surface [28], and at least one model of that zonal flow shows substantial variation in atmospheric angular momentum over the course of a Saturn year [29], and corresponding annual variations in the rotation rate of the solid surface [30]. However, for this mechanism to explain the entire apparent discrepancy, the moment of inertia of the participating solid part of Titan must be much less than that of the entire body, hence the inference of a decoupled shell [27]. Unfortunately, that analysis ignored both gravitational coupling to the deeper interior and gravitational torques from Saturn [29]. It appears unlikely that the atmosphere can cause sufficient change.

It is also not clear that there is any super-rotation effect to explain. To first order, a synchronous body has a rotation perod which matches the orbital period. If the orbit is inertially fixed, that is the whole story. However, if the orbit is precessing, then the spin rate must be somewhat faster in order to match the sum of orbital mean motion and apsidal precession rates. That is very nearly the case for Titan. We conclude that it is a synchronous rotator, as would be expected for a tidally damped body.

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